Reg. No.										

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

I SEMESTER B.S. DEGREE EXAMINATION – JUNE 2016

SUBJECT: MATHEMATICS -I (MA 111)

(COMMON TO ALL BRANCHES)

TUESDAY, 7th JUNE, 2016

Time: 3 Hours **Max. Marks: 100**

- **Answer ANY FIVE full Questions.**
- **Each main question carries (8 + 8 + 4) marks.**

1A) If $0 < a < b$ then, prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ and hence deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

1B) Obtain a reduction formula for $\int \cos^m x \sin^n x dx$ when m and n are non-negative integers. Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^m x \sin^n x dx$.

1C) Trace the curve $x = a\cos^3\theta, y = bsin^3\theta, a > b$ with explanations.

2A) Trace the curve $r = acos3\theta$ with explanations.

2B) If $y = e^{a\sin^{-1} x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$.

2C) Find the n^{th} derivatives of $\frac{x^2}{(x+2)(2x+3)}$

3A) The tangents at two points P, Q on the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - cos\theta)$ are at right angles. If ρ_1, ρ_2 are the radii of curvature at these points then show that $\rho_1^2 + \rho_2^2 = 16a^2$.

3B) Trace the curve $xy^2 = a^2(a - x)$ with explanations.

3C) Find the radius of curvature for the curve $r = a(1 + cos\theta)$.

4A) Find the area of the portion included between $r = a(1 + cos\theta)$ & $r = a(1 - cos\theta)$.

4B) Find the radius of curvature for the Folium of Descartes $x^3 + y^3 = 3axy$ at the point $(\frac{3a}{2}, \frac{3a}{2})$.

4C) Find the Equation of the plane passing through the point (1, 1, 3) and parallel to the plane $3x + 4y - 5z = 0$.

5A) Show that evolute of the curve $x = a\ (cost + \log\tan\frac{t}{2})$, $y = a\sin t$ is

$y = a\cosh\left(\frac{x}{a}\right)$.

5B) Find the volume of the solid generated by revolving the loop of the curve

$y^2(a - x) = x^2(a + x)\ , a > 0$ about x- axis.

5C) Test the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots\infty$.

6A) Find the image (reflection) of the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{4}$ in the plane $2x + y + z = -2$.

6B) State the values of x for which the following series converge

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots\infty$.

6C) Show that the planes $2x + 3y - z + 4 = 0\ and\ 4x + 6y - 2z - 5 = 0$ are parallel. Find the perpendicular distance between them.

7A) Expand $\tan^{-1} x$ in powers of $(x - 1)$ up to third degree terms by Taylor's theorem.

7B) Find the equation of the right circular cylinder having the circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$ as base circle.

7C) Discuss the convergence of the series $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \frac{5^5x^5}{5!} + \cdots\infty$.

8A) Find the line through the point $(2, -3\ , -4)$ which intersects the lines

$\frac{x+2}{2} = \frac{y-1}{-1} = \frac{z+2}{-2}$ & $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{3}$.

8B) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$.

8C) Find the point where the line $\frac{x-2}{2} = \frac{y-4}{-3} = \frac{z+6}{4}$ meets the plane $2x + 4y - z - 2 = 0$.