Reg. No.										

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
(Manipal University)
II SEMESTER B.S. DEGREE EXAMINATION – JUNE 2016
SUBJECT: MATHEMATICS -II (MA 121)
(COMMON TO ALL BRANCHES)
MONDAY, 6th JUNE, 2016

Time: 3 Hours **Max. Marks: 100**

- **Answer ANY FIVE full Questions.**
- **Draw diagrams and equations whenever necessary.**

1A. If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3} + log\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

1B. Evaluate $\iint_R x^2 dxdy$ where R is the region in the first quadrant bounded by the lines $y = x, y = 0, x = 8$ and the curve $xy = 16$.

1C. Define linearly dependent and independent vectors and give one example for each.

(8+8+4 = 20 marks)

2A. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane.

2B. Using the transformation $x + y = u \; and \; y = uv$, evaluate $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dydx$.

2C. Show that $\tau\left(\frac{1}{2}\right) = \sqrt{\pi}$. Hence find $\tau\left(\frac{-7}{2}\right)$.

(8+8+4 = 20 marks)

3A. i) If $u = f(x - y, y - z, z - x)$, Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

ii) If $u = xlogxy$ where $x^3 + y^3 + 3xy = 1$. Find $\frac{du}{dx}$.

3B. Show that $\beta(m, n) = \frac{\tau(m)\tau(n)}{\tau(m+n)}$, for $m, n > 0$.

3C. Using Gauss Jordan method, find the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

[(5+3) +8+4 = 20 marks]

4A. Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$.

4B. Evaluate $\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \frac{dzdydx}{(1+x^2+y^2+z^2)^2}$.

4C. In estimating the cost of a pile of bricks measured as $2m \times 15m \times 1.2m$, the tape is stretched 1% beyond the standard length. If the count is 450 bricks $to\ 1cu.m$ and bricks cost $Rs. 530\ \text{per}\ 1000$, find the approximate error in the cost.

(8+8+4 = 20 marks)

5A. i) Determine the constant a so that the following vector is solenoidal.

$\vec{V} = (-4x - 6y + 3z)\hat{\imath} + (-2x + y - 5z)\hat{\jmath} + (5x + 6y + az)\hat{k}$.

ii) Find constants $a, b, c\ so\ that\ \vec{V} = (-4x - 3y + az)\hat{\imath} + (bx + 3y + 5z)\hat{\jmath} + (4x + cy + 3z)\hat{k}$ is irrotational. Hence show that $\vec{V}$ can be expressed as the gradient of a scalar function.

5B. Show that $\int_0^{\infty} \sqrt{y}e^{-y^2}dy \times \int_0^{\infty} \frac{e^{-y^2}}{\sqrt{y}} dy = \frac{\pi}{2\sqrt{2}}$.

5C. The diameter and altitude of a 'can' in the shape of the right circular cylinder are measured as $40cm$ and $64cm$ respectively. The possible error in each measurement is $\mp 5\%$. Find approximately the maximum possible error in the computed value for the volume and the lateral surface. Find the corresponding percentage error.

[(2+6)+8+4 = 20 marks]

6A. Find the extreme values of $f(x, y) = xy(a - x - y)$.

6B. Construct orthonormal basis from the following set of vectors

$\{(1, 1, 1), (-1, 0, -1), (-1, 2, 3)\}$.

6C. Find the area between the curve $r = a(sec\theta + cos\theta)$ and the asymptote $\mathrm{r} = \mathrm{a}sec\theta$.

(8+8+4 = 20 marks)

7A. A rectangular box open at the top have volume $32\ cubic\ feet$. Find the dimension of the box requiring least material for constructions.

7B. i) Define Rank of the matrix.

ii) Find the values of a and b for which the following system

$$x - y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b.$$

have a) Unique solution.

b) Infinitely many solutions
c) No solution

7C. Show that $\nabla\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = c$.

[8+ (1+7) +4 = 20 marks]

8A. Find the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

8B. Find the volume inside the cone $x^2 + y^2 = z^2$ bounded by the sphere $x^2 + y^2 + z^2 = a^2$.

8C. If x increases at the rate of $2cm/sec$ at the instant when $x = 3cm$ and $y = 1cm$.

At what rate must y be changing in order that the function $2xy - 3x^2y$ shall be neither

Increasing nor decreasing?

(8+8+4 = 20 marks)