Reg. No.										

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
(Manipal University)
III SEMESTER B.S. DEGREE EXAMINATION – MAY 2016
SUBJECT: MATHEMATICS -III (MA 231)
17TH MAY, 2016

Time: 3 Hours **Max. Marks: 100**

- **Answer ANY FIVE full Questions.**
- **Draw diagrams and equations whenever necessary.**

1A. i) Find $L[te^{-2t}sin4t]$, ii) Find $L\left[\frac{2sintsin5t}{t}\right]$.

1B. Solve $(x^2 - y^2)dx - xydy = 0$.

1C. Form differential equation of all circles with center at (h, k) and radius a.

(8+8+4 = 20 marks)

2A. Solve $y'^{v} + 2y'' + y = x^2cosx$.

2B. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.

2C. If $w = \emptyset + i\varphi$ represents the complex potential for an electric field and

$\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function $\emptyset$. **(8+8+4 = 20 marks)**

3A. Find by Taylor's series method the value of y at $x = 0.2$ for the differential equation

$\frac{dy}{dx} = 2y + 3e^x,\ y(0) = 0$. Compare the solution obtained with the exact solution.

3B. Show that polar form of Cauchy-Riemann equations are

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$,$\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Hence deduce $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

3C. Solve $y'' - 4y = \cosh(2x - 1)$. **(8+8+4 = 20 marks)**

4A. Express the following function in terms of unit step function and hence find it's Laplace

transform. Also find $L[e^{-t}(1 - u(t - 2)]$. $f(t) = \begin{cases}(t-1) & ; \quad 1 < t < 2 \\ (3-t) & ; \quad 2 < t < 3\end{cases}$

4B. Evaluate

i) $\oint_C \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3} dz$, where C is the circle $|z| = 1$,

ii) $\oint_C \frac{e^{2z}}{(z+i)^4} dz, where\ C\ is\ the\ circle\ |z| = 3.$

4C. Using Modified Euler's method, find an approximate value of y when $x = 0.2$, given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$, taking $h = 0.1$ **(8+8+4 = 20 marks)**

5A. Solve the simultaneous equation $\frac{dx}{dt} = 7x - y$; $\frac{dy}{dt} = 2x + 5y$.

5B. Apply Runge-Kutta method of order 4 to find an approximate value of y for $x = 0.2$ in steps of 0.1 $if \frac{dy}{dx} = x + y^2$, given that $y = 1$ when $x = 0$

5C. Find $L[(1 - e^{2t})u(t-1)]$ **(8+8+4 = 20 marks)**

6A. Solve $x^2y'' + xy' + 9y = 3x^2 + \sin(3logx)$.

6B. Solve by the method of transforms, the equation $\frac{d^2x}{dt^2} + 9x = cos2t$, given $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$.

6C. Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at it's poles. **(8+8+4 = 20 marks)**

7A. Solve $y'' + 4y = tan2x$ using the method of variation of parameters.

7B. Define Periodic function. Draw the graph of the periodic function.

$$f(t) = \begin{cases} t\ ; 0 < t < \pi \\ \pi - t\ ;\ \pi < t < 2\pi \end{cases}$$ Hence find Laplace transform of $f(t)$.

7C. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$. **(8+8+4 = 20 marks)**

8A. Solve $2y'cosx + 4ysinx = sin2x$, given that $y = 0$ $x = \frac{\pi}{3}$.

8B. i) Find $L^{-1}\left[\frac{s^2}{(s-2)^3}\right]$, (ii) Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$.

8C. Form the partial differential equations by eliminating arbitrary functions from $f(x^2 + y^2, z - xy) = 0$. **(8+8+4 = 20 marks)**