

Time: 3 Hours

Max. Marks: 100

 $(4 \times 5 = 20M)$

Answer any FIVE questions from each part.

<u>PART - A</u> (2×5=10M)

- 1A. If A and B are two mutually exclusive events of S then $P(A \cup B) =$
- 1B. A pair of dice are thrown. What is the probability that sum is greater than 8?
- 1C. The probability density function of exponential distribution is _.
- 1D. If A and B are two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$, then find P(A/B).
- 1E. Classify the following PDE : $u_{xx} + u_{xy} = f(x, y)$
- 1F. Express f(x) = x as a Fourier series in the interval $-\pi < x < \pi$.
- 1G. Suppose a random variable X:0,1,2 has its probabilities $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{2}$ respectively. Find cdf.

<u> PART - B</u>

2A. A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen at random from the lot. Find the probability thata) both are good b) atleast one is good c) at most one is good.

- 2B. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that a) exactly 2 will be defective b) atleast 2 will be defective.
- 2C. Fit a straight line to the following data

Х	61	71	81	91	101
Y	8	10	12	10	16

2D. If A and B are two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$ then find a) $P(A \cap \overline{B})$ b) $P(A/\overline{B})$ c) P(A/B)

- 2E. Suppose that a breaking strength of cotton say X is normally distributed with $\mu = 165$ and $\sigma^2 = 9$. Assume that a sample of this fabric is considered to be defective if x < 162. What is the probability that a fabric chosen at random will be defective?
- 2F. Find the half range cosine series for the function $f(x) = x^2$ in the range $0 \le x \le \pi$.
- 2G. The odds that a person X speaks the truth are in the ratio 3:2 and the person Y speaking the truth is in the ratio 5:3. In what percentage of cases are they likely to contradict each other on an identical point?

<u>PART - C</u> (6×5=30M)

- 3A. The chance that a doctor A will diagnose the disease correctly is 60%. Chance that patient A die after proper diagnosis is 40%. Chance that patient of A will die after wrong diagnosis is 70%. If patient of A dies, what is the probability that his disease was correctly diagnosed?
- 3B. Suppose a random variable X has pdf $f(x) = \begin{bmatrix} k(1-x^2), & 0 < x < 1 \\ 0, & \text{otherwise} \end{bmatrix}$. Then find

a) the value of k b) mean and variance of X.

- 3C. The probability that an individual suffers from a bad reaction is 0.001. Determine the probability that out of 2000 individualsa) exactly 3, b) more than 2, individuals suffer from a bad reaction.
- 3D. If $f(x) = |\sin x|$, expand f(x) as a Fourier series in $(-\pi, \pi)$.
- 3E. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$ taking h = 1 upto t = 0.75. The boundary conditions are u(0,t) = u(5,t) = 0, $u_i(x,0) = 0$ and $u(x,0) = x^2(5-x)$.
- 3F. Find the Fourier series to represent the function f(x) by $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi x, & \pi \le x \le 2\pi \end{cases}$

3G. Compute an approximate probability that a mean sample of size 15 from a distribution having pdf $f(x) = \begin{bmatrix} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{bmatrix}$ is between $\frac{3}{5}$ and $\frac{4}{5}$.

PART - D

 $(8 \times 5 = 40M)$

4A. Find the joint distribution of X and Y which are independent random variables with the following respective distributions.

Х	1	2	У	-2	5	8
f(x)	0.7	0.3	g(y)	0.3	0.5	0.2

Also find Cov(X, Y).

4B. Suppose that 3 uncorrelated random variables X, Y, Z has standard deviation 5, 12, 9 respectively. Find correlation coefficient between U = X + Y and V = Y + Z.

4C. If
$$f(x) = \left(\frac{\pi - x}{2}\right)^2$$
 in the range 0 to 2π , show that $f(x) = \frac{\pi^2}{12} + \sum_{1}^{\infty} \frac{\cos nx}{n^2}$.

- 4D. Solve the PDE $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides x = 0 = y, x = 3 = y with u = 0 on the boundary and mesh length 1.
- 4E. An insurance company has discovered that only about 0.1% of the population is involved in certain type accident in each year. If its 10000 policy holders were randomly selected from the population, what is the probability that not more than 5 are involved in such an accident the next year?
- 4F. Suppose that a two-dimensional continuous random variable has a pdf

$$f(x,y) = \begin{bmatrix} \frac{x^2 + xy}{3}, & 0 \le x \le 1, 0 \le y \le 2\\ 0, & \text{elsewhere} \end{bmatrix}.$$

Show that f(x, y) is a joint pdf. Also find P(X+Y < 1).

4G. Let a random sample of size 17 from $N(\mu, \sigma^2)$ yields $\overline{X} = 4.7$ and $s^2 = 5.76$. Determine 90% confidence interval for μ .

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