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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

IV SEMESTER B.E DEGREE END SEMESTER EXAMINATION–MAY 2016

SUB: ENGG.MATHEMATICS – IV (MAT – 2208)

Duration: 3 Hrs. **Max. Marks: 50**

✍ **Note: a) Answer all questions. b) All questions carry 10 (4 + 3 + 3) marks.**

1A. Urn A contains 8 white and 7 black balls. Urn B contains 9 black and 7 white balls. A ball is randomly drawn from Urn A and placed in B and then a ball is transferred from Urn B to Urn A. Finally a ball is selected from Urn A. What is the probability that the ball is white?

1B. Solve$(E^3 - 5E^2 + 3E + 9)y = 2^x + 3^x$.

1C. If X is a random variable taking the values 0,1,2,… and $P(x) = ab^x$ where a and b are positive numbers such that $a + b = 1$, find the moment generating function of X. If $E(X) = m_1$ and $E(X^2) = m_2$, show that $m_2 = m_1(1 + 2m_1)$.

2A. Suppose that the life-lengths of two electronic devices, say D_1 and D_2 have distributions $N(40,36)$ and $N(45,9)$ respectively. If the device is to be used for 45-hour period, which device is to be preferred? If it is to be used for a 48-hour period, which device is to be preferred?

2B. With $h = 1$, solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10), 0 < x < 3, 0 < y < 3$. Given $u = 0$ on the boundary.

2C. Prove that n straight lines in a plane, no two parallel, no three concurrent, divide the plane into $\frac{1}{2}(n^2 + n + 2)$ parts.

3A. If $f(x,y) = \begin{cases} kx(x-y), 0 < x < 2, -x < y < x \\ 0, \quad elsewhere \end{cases}$ is a p.d.f., find k. Find the marginal p.d.f of X and Y.

3B. A two dimensional random variable (X, Y) is uniformly distributed in the region bounded by a circle $x^2 + y^2 = a^2$. Find the correlation coefficient between X and Y.

3C. Diameter of an electric cable is assumed to be a continuous random variable with p.d.f

$$f(x, y) = \begin{cases} 6x(1-x), 0 < x < 1 \\ 0 \quad , \quad elsewhere \end{cases}.$$

(i) Determine the Cumulative distribution function of $f(x)$.

(ii) Compute $P(X \leq 1/2 \mid 1/3 \leq X \leq 2/3)$.

4A. Solve by Z-transform method: $y_{n+2} + 2y_{n+1} + y_n = n, y_0 = y_1 = 0$.

4B. A bag contains 3 coins, one of which is coined with two heads and the other two coins are normal and not biased. A coin is chosen at random from the bag and tossed four times in succession. If head turns up each time, what is the probability that it is a two-headed coin?

4C. A computer, in adding numbers, rounds each number off to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (-0.5, 0.5). If 1500 numbers are added, what is the probability that the magnitude of the total error exceeds 15?

5A. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ $, 0 < x < 2, t > 0. u(x, 0) = 0, \frac{\partial u}{\partial t}(x, 0) = 100(2x - x^2)$

$u(0, t) = u(2, t) = 0$. Take $h = \frac{1}{2}$ and compute the solution for four levels.

5B. Let $X_1, X_2, \dots X_n$ be n independent random variables with distributions $N(\mu_1, \sigma_1^2), N(\mu_2, \sigma_2^2), \dots, N(\mu_n, \sigma_n^2)$, respectively. Find the distribution of

$Y = K_1X_1 + K_2X_2 + \cdots + K_nX_n$ and hence establish the reproductive property of normal distribution.

5C. Let X be a continuous random variable with pdf given by,

$f(x) = \frac{1}{\pi(1+x^2)}; -\infty < x < \infty$. Find the distribution of $Y = \frac{1}{X}$.

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