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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

IV SEMESTER B.Tech DEGREE END SEMESTER EXAMINATION

SUB: ENGG. MATHEMATICS – IV (MAT – 2211)
(REVISED CREDIT SYSTEM)

Duration: 3 Hrs. **Max. Marks: 50**

✍ **Note: a) Answer the following questions. b) All questions carry 10 (4 + 3 + 3) marks**.

1A. Prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = \begin{cases} 0 & for\ \alpha \neq \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2 & for\ \alpha = \beta \end{cases}$ where $\alpha, \beta\ are\ the\ roots\ of\ J_n(x) = 0$

1B. N men throw their hat in the center of a hall and then each one picks a hat at random. What is the probability that no one gets his own hat?

1C. Let A and B are two events associated with an experiment. Given P(A)=0.4, P(A∪ B)=0.7 and P(B)=p. For what choice of p i) A and B are independent
ii) A and B are mutually exclusive

2A. State Central limit theorem. Hence compute the probability $P(49 < \overline{X} < 51)$, where $\overline{X}$ denotes the mean of the sample space of size 100 from a distribution which has $\chi^2(50)$.

2B. An officer is in hurry to reach airport to catch the flight scheduled at 1AM. Probability that he gets a taxi at such an early hour is 0.23. However if he gets a taxi, he would catch flight with probability 0.85. If he doesn't get a taxi, he will catch the flight with probability 0.43 by some other mode of transportation. If he catches the flight, what is the probability that he came by taxi?

2C. Prove that $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$

3A. Solve in series $(1-x^2)y'' - xy' + 4y = 0$

3B. Suppose that 2- dimensional continuous random variable has joint pdf given by

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3} & 0 \le x \le 1, 0 \le y \le 2 \\ 0 & elsewhere \end{cases}$$

Find i) V(X)

ii) $P(X+Y \ge 1)$

3C If $\overline{X}$ be the sample mean of random sample size n from $N(\mu, 100)$ distribution. Find n such that $P(\mu - 5 < \overline{X} < \mu + 5) = 0.954$.

4A. Suppose that the random variable X has pdf given by $f(x) = \frac{e^{-|x|}}{2}, -\infty < x < \infty$. Find the moment generating function of X and hence find $E(X)$ *and* $V(X)$.

4 B Solve the difference equation $y_{n+2} - 2y_{n+1} + y_n = n^2 2^n$.

4C. Fit a second degree parabola for the following data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

5A. . Find Z-transform of the following

i. $\sin(n\theta)$

ii. $a^n \sin(n\theta)$

iii. $\sin(3n + 5)$

5B. Suppose that 2 dimensional random variable (X,Y) is uniformly distributed over the triangular region R={(x,y)| $0 < x < y < 1$} Find its i)pdf and ii)ρ

5C. (i) The diameter of an electric cable is normally distributed with mean 0.8 and variance 0.0004. What is the probability that the diameter will exceed 0.81 inches?

(ii) If X has the pdf $f(x) = 1, \quad 0 \le x \le 1,$ Find the pdf of $Y = -2logX$.