Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)

प्रज्ञानं ब्रह्म
Manipal
INSPIRED BY LIFE

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IV SEMESTER B.TECH (INDUSTRIAL & PRODUCTION ENGINEERING)
MAKEUP EXAMINATIONS, JUNE 2016

SUBJECT: ENGINEERING   MATHEMATICS IV (MAT 2209)
REVISED CREDIT SYSTEM

Time: 3 Hours   MAX. MARKS: 50

Instructions to Candidates

❖ Answer **ALL** the questions. All questions carry equal marks

1A.	Fit a second degree parabola to the following data: x: 1 2 3 4 5 y: 10 12 13 16 19	**3**
1B.	Ten persons in a room are wearing badges numbered 1 through 10. Three persons are selected at random and are asked to leave the room simultaneously. Their badge numbers are noted. What is the probability that the (i) smallest badge number is 5 (ii) largest badge number is 5?	**3**
1C.	Find the correlation coefficient and the regression lines of y on x and x on y for the following data. <table><tr><td>x</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td></tr><tr><td>y</td><td>16</td><td>10</td><td>8</td><td>20</td><td>5</td></tr></table>	**4**
2A.	Two independent random variables X_1 and X_2 have means 5, 10 and variance 4, 9. Find the covariance between $U = 3X_1 + 4X_2$ and $V = 3X_1 - X_2$.	**3**
2B.	A continuous random variable has the pdf $f(x) = \begin{cases} kx^3, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ Find (i) the constant k (ii) $\Pr\left\{\frac{1}{4} < X < \frac{3}{4}\right\}$ (iii) $\Pr\{X < \frac{1}{2}\}$.	**3**

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2C.	The joint pdf of a two dimensional random variable is given by $$f(x, y) = x^2 + \frac{xy}{3}, \quad 0 < x < 1, \quad 0 < y < 2$$ $$= 0, \qquad\qquad \text{elsewhere}$$ Compute (i) $P\{X + Y \geq 1\}$ (ii) $P\{Y < 1/2 \mid X < 1/2\}$.	**4**
3A.	In a bolt factory there are three machines A, B, C manufacturing respectively 40%, 25%, 35% of the total production. Of these 5%, 3% & 2 % are defective. If a bolt is drawn at random was found to be defective, what is the probability that it was manufactured by A?	**3**
3B.	Suppose that X has Poisson distribution. If $Pr\{X = 2\} = \frac{2}{3} Pr\{X = 1\}$. Evaluate (i) $Pr\{X=0\}$, (ii) $Pr\{X = 3\}$.	**3**
3C.	If $X \sim N(\mu, \sigma^2)$ show that $W = \left(\dfrac{X - \mu}{\sigma}\right)^2$ has $\chi^2(1)$.	**4**
4A.	Find the mean and variance of Chi-square distribution.	**3**
4B.	If X and Y are independent and have standardized normal distribution, show that $Z = \frac{X}{Y}$ has Cauchy's distribution.	**3**
4C.	In a normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and variance of the distribution.	**4**
5A.	Find the mgf of a random variable which is uniformly distributed over (-a, a). Hence evaluate $E(X^{2n})$.	**3**
5B.	Suppose that a continuous random variable X has the pdf $f(x) = e^{-x}$, $x \geq 0$. Find the pdf of $Y = X^3$.	**3**
5C.	A random sample of size 64 is taken from an infinite population having mean 112 and variance 144. Using central limit theorem, find the probability of getting $\bar{x} > 114.5$	**4**