Reg. No.

# Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)

## IV SEMESTER B.TECH (MECHANICAL ENGINEERING)

## END SEMESTER EXAMINATIONS, MAY 2016

## SUBJECT: ENGINEERING MATHEMATICS IV [MAT 2210]

**REVISED CREDIT SYSTEM**

Time: 3 Hours MAX. MARKS: 50

**Instructions to Candidates:**

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

**1A.** The p.d.f. of a continuous random variable $X$ is given by

$$f(x)=\begin{cases}6x(1-x) & \text{for } 0\le x\le 1\\ 0 & \text{elsewhere}\end{cases}$$

then (i) find the c.d.f. of $X$.
(ii) find $E(X)$ and $V(X)$. **4**

**1B.** In a game A and B throw a pair of dice alternatively. A wins if he throws a sum 6 before B throws a sum 7 and B wins if he throws a sum 7 before A throws a sum 6. If A begins then find his chance of winning. **3**

**1C.** Three urns contain 6 red, 4 black; 4 red, 6 black and 5 red, 5 black balls respectively. One of the urns is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first urn? **3**

**2A.** Fit a parabola $y=a+bx+cx^2$ to the following data **4**

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 1 | 5 | 10 | 22 | 38 |

**2B.** Find the moment generating function (m.g.f.) of a Poisson distribution and hence find its mean and variance. **3**

**2C.** A two dimensional continuous random variable (X, Y) has the joint p.d.f.

$$f(x,y)=\begin{cases}kxy & ;\ 0<x<4, 1<y<5\\ 0 & ;\ \text{elsewhere}\end{cases}$$

Then find $k$ and $E(XY)$. **3**

| | | |
|---|---|---|
| **3A.** | The marks of a set of students for a certain subject are normally distributed with mean 62 and standard deviation 3. If 4 students are chosen randomly from this set then what is the probability that 3 of them have less than 60% marks. | **4** |
| **3B.** | In a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Using binomial distribution, how many would be expected to contain at least 3 defective parts out of 1000 such samples. | **3** |
| **3C.** | Let $X$ be random variable which is uniformly distributed over $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$. Then find the p.d.f. of $Y=\tan X$. | **3** |
| **4A.** | If $X_1$ and $X_2$ are two independent random variables which follows standard normal distribution then find the p.d.f. of $Y=\frac{X_1}{X_2}$. | **4** |
| **4B.** | Let $\mu$ and $\sigma^2$ be the mean and variance of a continuous random variable $X$ with p.d.f. $f(x)$ then prove that $P(\mu-k\sigma<X<\mu+k\sigma)\geq 1-\frac{1}{k^2}$ for any $k>0$. | **3** |
| **4C.** | Express $f(x)=2x^3-x^2-3x+2$ in terms of Legendre polynomials. | **3** |
| **5A.** | Prove that<br>$(i)\frac{d}{dx}\left[x^n J_n(x)\right]=x^n J_{n-1}(x)\quad(ii)\,xJ_n^{\,1}(x)=xJ_{n-1}(x)-nJ_n(x)$ | **4** |
| **5B.** | Obtain the power series solution of the equation $\frac{d^2y}{dx^2}-y=0$ | **3** |
| **5C.** | Let $\overline{X}$ be the mean of a random sample of size 15 from a distribution having p.d.f. $f(x)=\begin{cases}3x^2 & \text{for } 0<x<1\\ 0 & \text{elsewhere}\end{cases}$. Then by using central limit theorem find $P\left(\frac{3}{5}<\overline{X}<\frac{4}{5}\right)$. | **3** |