Reg.No



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IV SEMESTER B.E DEGREE END SEMESTER MAKEUP EXAMINATION–JULY, 2016 SUB: ENGG.MATHEMATICS – IV (MAT – 2208)

Duration: 3 Hrs.

Max. Marks: 50

X Note: a) Answer all FIVE full questions. b) All questions carry 10 (4 + 3 + 3) marks.

1A. Solve $(E^2 - 2E + 2)y = \cos\left(\frac{n\pi}{2}\right)$.

- 1B. Consider a family of 'n' children. Let A be an event that the family has children of both the sexes, and B be the event that there is at most one girl in the family. Find the value of n for which A and B are independent, assuming that each child has the probability $\frac{1}{2}$ of being a boy.
- 1C. Find the moment generating function of Normal distribution.
- 2A. Suppose that the joint p.d.f. of (X,Y) is given by $f(x,y) = \begin{cases} e^{-y}; & x > 0, y > x \\ 0; & otherwise \end{cases}$. Then find (i) marginal pdf of X, (ii) marginal pdf of Y (iii) $\Pr(X > 2|Y < 4)$.
- 2B. Consider a random variable X with possible outcomes 0,1,2,.... Suppose that $Pr(X = j) = (1 a)a^j, j = 0,1,2,...,$
- (i) find the values of a for which the above is a valid probability distribution
- (ii) Show that for any two positive integers s and t, $Pr(X > s + t | X > s) = Pr(X \ge t)$.
- 2C. Solve by Z-transform method

 $y_{n+2} + 6y_{n+1} + 9y_n = 2^n, y_0 = y_1 = 0.$

3A. Use implicit method and solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$, 0 < x < 2, t > 0 such that

 $u(x,0) = 0, u(0,t) = \sin\left(\frac{8\pi t}{3}\right), u(2,t) = 0.$ Compute u(x,t) for two time levels by taking h = 0.5.

- 3B. The annual rainfall at a certain locality is known to be normally distributed random variable with mean 29.5 inches and standard deviation 2.5 inches. How many inches of rain (annually) is exuded about 5 percent of the time?
- 3C. While playing with an opponent of equal ability which of the following is more probable?
 - (i) Winning 3 games out of 4 or 5 games out of 8?
 - (ii) Winning at least 3 games out of 4 or at least 5 games out of 8?

4A. Compute approximately the probability that the mean of the random sample of size

15 from a distribution having pdf $f(x) = \begin{cases} 3x^2; & 0 < x < 1\\ 0; & elsewhere \end{cases}$ is between $\frac{3}{5}$ and $\frac{4}{5}$.

4B. Solve
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0, u(x, 0) = 1 - x^2, u(0, t) = 1 - t^2, \frac{\partial u}{\partial t}(x, 0) = 0,$$

u(1, t) = 0. Take h=0.25 and compute the solution for 2 levels.

4C. Find the inverse Z-transform of (i)
$$\frac{z^2+z}{(z-1)(z^2+2)}$$
 (ii). $\frac{z}{z^2+7z+10}$

5A. Three boxes B1, B2 and B3 contain white(W), black(B) and red(R) balls as follows:

A die is rolled and B1 is selected if the number is 1 or 2, B2 if the number is 3 or 4 and B3 if the number is 5 or 6. If a ball is drawn from the box thus selected, find the probability that the ball is from the box B2 being a black ball.

5B. With h=0.5, Compute the solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, |x| < 1, |y| < 1

$$u(x,\pm 1) = x^2, u(\pm 1, y) = y^2.$$

5C. Two independent random variables X_1, X_2 have mean (5, 10) and variance (4, 9)

respectively. Find the correlation coefficient between $u = 3X_1 + 4X_2$ and $v = 3X_1 - X_2$.
