Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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SECOND SEMESTER M.Tech. (CS) DEGREE END SEMESTER EXAMINATION
May/June 2016
SUBJECT: OPTIMAL CONTROL (ICE - 524)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates

- Answer **ANY FIVE** full questions.
- Missing data may be suitably assumed.

1A. Find the optimum of the functional $J(x) = \int_0^{\pi/2}[\dot{x}^2(t) + x^2(t)]dt$ which satisfy the boundary condition $x(0) = 0$ and $x(\pi/2) = 1$

1B. Define variation and optimum of functional. Also state the fundamental theorem of calculus of variation.

1C. Illustrate the performance index for minimum-energy control system and fuel optimal control system.

(5+3+2)

2A. Write the statement of Bolza Problem and briefly illustrate the solution procedure using Pontryagin principle.

2B. Given a second order (double integrator) system as

$$\dot{x}_1(t) = x_2(t), \qquad \dot{x}_2(t) = u(t),$$

and the performance index as $J = \frac{1}{2}\int_{t_0}^{t_f} u^2(t)dt$. Find the optimal control and optimal state for the given boundary condition (initial and final) condition $X(0) = [1\ 2]^T$; $X(2) = [1\ 0]^T$. Assume that the control and states are unconstrained

(5+5)

3A. Consider a simple first order system $\dot{x}(t) = -3x(t) + u(t)$ and the performance measure as $J = \int_0^{\infty}[x^2(t)+u^2(t)]dt$, where $x(0) = 1$ and the final state $x(\infty) = 0$. Find the closed loop optimal controller.

3B. Given plant as $\dot{x}(t) = ax(t) + bu(t),$ performance index $J = \frac{1}{2}\int_{t_o}^{t_f}[qx^2(t) + ru^2(t)]dt$ and boundary condition as $x(t = 0) = x_0$, $x\left(t = t_f\right) = 0$. Find the closed-loop optimal control.

(5+5)

4A. Establish Hamilton-Jacobi-Bellman equation for the plant $\dot{x}(t) = f(x(t), u(t), t)$ with performance index $J(x(t_0), t_0) = \int_{t_0}^{t_f} V(x(t), u(t), t)dt$.

4B Consider a simple first order system

$$\dot{x}(t) = -x(t) + u(t), \qquad x(0) = 1,$$

and cost function as $\boldsymbol{J = \int_0^\infty e^{2\alpha t}[x^2(t) + u^2(t)]dt.}$ Find the optimal control law and show that the closed-loop optimal control system has a degree of stability of at least α

(5+5)

5A. Given a second order plant

$$\dot{x}_1(t) = x_2(t), \qquad x_1(0) = 2$$

$$\dot{x}_2(t) = -2x_1(t) + x_2(t) + u(t), \qquad x_2(0) = -3$$

and the performance index $J = \frac{1}{2}\int_0^\infty \left[2x_1^2(t) + 6x_1(t)x_2(t) + 5x_2^2(t) + 0.25u^2(t)\right] dt$ **,** obtain the optimal feedback control law.

5B. The system

$$\dot{x}_1(t) = x_2(t),$$

$$\dot{x}_2(t) = -x_1(t) + \left[1 - x_1^2(t)\right]x_2(t) + u(t),$$

is to be controlled to minimize the performance measure $J = \int_0^1 \frac{1}{2}\left[2x_1^2(t) + x_2^2(t) + u^2(t)\right]dt$. The initial and final states values are specified. Determine the co-state equations and control that minimizes the Hamiltonian for $u(t)$ not bounded.

(5+5)

6A. Consider a simple pendulum in a vertical plane which is governed by

$$\dot{x}_1(t) = x_2(t), \quad x_1(0) = -1.5$$

$$\dot{x}_2(t) = -4x_2(t) + 6u(t), \quad x_2(0) = 0$$

It is required to return the system to its equilibrium state in minimum time. Find the optimal control law.

6B. The simple motion of an inertial load in the frictionless environment is described by

$$m\ddot{y}(t) = f(t)$$

where, m is the mass of a body, $y(t)$ is the position $f(t)$ is the external force applied to the system. Formulate the time optimal control problem and write the problem statement for this system.

6C. Define signum function with neat diagram.

(5+3+2)