

MANIPAL INSTITUTE OF TECHNOLOGY, MANIPAL 576104 (Constituent College of Manipal University)



SIXTH SEMESTER B.TECH DEGREE MAKE UP EXAMINATION-JULY 2016 SUBJECT:OPEN ELECIVE-II MACHINE LEARNING (ICT 364) (REVISED CREDIT SYSTEM)

TIME: 3 HOURS

8/07/2016

MAX. MARKS: 50

Instructions to candidates

- Answer any FIVE FULL questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.
- 1A. With probabilistic assumption and interpretation, derive the required relation for least-square regression.
- 1B. What are the limitations of linear regression? How does locally weighted linear regression overcome those limitations?
- 1C. A generalized linear model assumes that the response variable y (conditioned on x) is distributed according to a member of the exponential family:

$$p(y; \eta) = b(y) \exp(\eta^T T(y) - a(\eta)).$$

Show that the Gaussian distribution is an example of exponential distribution.

[5+3+2]

2A. Suppose you are given a dataset $\{(x^{(i)}, y^{(i)}; i = 1, ..., m)\}$ consisting of m independent examples, where $x^{(i)} \in \mathbb{R}^n$ are n-dimensional vectors, and $y^{(i)} \in \{0, 1\}$. You will model the joint distribution of (x, y) according to:

$$p(y) = \phi^{y} (1 - \phi)^{1-y}$$

$$p(x|y=0) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} (x - \mu_0)^T \Sigma^{-1} (x - \mu_0)\right)$$

$$p(x|y=1) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} (x - \mu_1)^T \Sigma^{-1} (x - \mu_1)\right)$$

Suppose you have already fit ϕ , μ_0 , μ_1 , and Σ , and now want to make a prediction at some new query point x. Show that the posterior distribution of the label at x takes the form of a logistic function, and can be written as

$$p(y = 1|x; \phi, \Sigma, \mu_0, \mu_1) = \frac{1}{1 + \exp(-\theta^T x)},$$

where θ is some appropriate function of $\phi, \Sigma, \mu_0, \mu_1$.

- 2B. What is the intuition behind margins in SVMs? Explain the following margins:
 - i) Functional margin, and
 - ii) Geometric margin.
- 2C. State Hoeffding inequality and give its interpretation.

[5+3+2]

Use EM algorithm to derive the expression for μ .

4C. Write k-means clustering algorithm.

$$[5+3+2]$$

- 5A. Consider a learning problem in which you have a finite hypothesis class $\mathcal{H} = \{h_1, \dots, h_k\}$ consisting of k hypothesis. Derive uniform convergence result.
- 5B. State Jensen's inequality and graphically depict its behavior.
- 5C. State Mercer's theorem on valid kernels.

[5+3+2]

6A. For the data set given in Table Q.6A, design a polynomial learning machines whose inner product kernel is given by

$$K(\mathbf{X}, \mathbf{X}_i) = (1 + \mathbf{X}^T \mathbf{X}_i)^2.$$

Table: Q.6A

Input Vector, x	Desired Response, d
(-1, -1)	-1
(-1, +1)	+1
(+1, -1)	+1
(+1, +1)	-1

- 6B. Consider a classification problem in which the response variable y can take on any one of k values, so $y \in \{1, 2, ..., k\}$. You can model such distribution as a multinomial distribution. Use GLM for modeling this multinomial distribution and derive the relation for parameters of exponential family distribution.
- 6C. Briefly explain Fisher scoring for maximizing $l(\theta)$.

[5+3+2]