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MANIPAL INSTITUTE OF TECHNOLOGY, MANIPAL 576104 (Constituent College of Manipal University)



## SIXTH SEMESTER B.Tech. (IT & CCE) DEGREE END SEMESTER EXAMINATION MAY – 201 SUBJECT: ELECTIVE – I : STATISTICAL ANALYSIS AND APPLICATIONS – ICT 322 (REVISED CREDIT SYSTEM)

| TIME: 3 HO       | OURS   |          | 11/05/2016             |                |      |           | MAX. MARKS: 50 |        |          |       |       |           |
|------------------|--|----------|------------------------|----------------|------|-----------|----------------|--------|----------|-------|-------|-----------|
| Instructions (   | Instructions to candidates                     |          |                        |                |      |           |                |        |          |       |       |           |
| Answe            | Answer any <b>FIVE FULL</b> questions.         |          |                        |                |      |           |                |        |          |       |       |           |
| Missin           | Missing data, if any, may be suitably assumed. |          |                        |                |      |           |                |        |          |       |       |           |
| <b>1A.</b> Use t | he Chi   | -square  | test w                 | ith $\alpha =$ | 0.05 | to test   | whether        | the da | ta shown | below | are u | uniformly |
| distrib          | uted. G  | iven tha | at X <sup>2</sup> 0.05 | ,9=16.9        | )    |           |                |        |          |       |       |           |
| 0.99             | 0.90   | 0.80     | 0.82                   | 0.98           | 0.99 | 0.81      | 0.72           | 0.85   | 0.75     |       |       |           |
| 0.78             | 0.83   | 0.77     | 0.70                   | 0.95           | 0.86 | 0.73      | 0.91           | 0.71   | 0.88     |       |       |           |
| 0.75             | 0.95   | 0.97     | 0.82                   | 0.78           | 0.83 | 0.94      | 0.01           | 0.10   | 0.15     |       |       |           |
| 1D A             |  |          |                        | .1             | 1 00 | 0/ - £ 41 |                | .1     |          |       |       | 1 1       |

- 1B. An agriculture cooperative claims that 90% of the watermelons shipped out are ripe and ready to
  - eat. Find the probabilities that among 18 watermelons shipped out
    - i) all 18 are ripe and ready to eat
    - ii) at least 16 are ripe and ready to eat
- 1C. Explain why there must be a mistake in each of the following statements
  - i) The probability that a student will get an A in a geology course is 0.32, and the probability that he or she will get either an A or a B is 0.27.

ii) A company is working on the construction of two shopping centers; the probability that the larger one will be completed on time is 0.35 and the probability that both will be completed on time is 0.42

## [5 + 3 + 2]

- **2A.** Generate 10 random numbers using multiplicative congruential generator with m=64, a = 13 and  $X_0 = 2$ . Also check whether they are independent using autocorrelation test. Given that  $Z_{0.025} = 1.96$ .
- **2B.** Service time at a cashier's window is normally distributed with mean 7.3 minutes and variance 11.7 minutes<sup>2</sup>. Generate a service time using the random numbers given below,

| 0.1758 | 0.1489 | 0.2774 | 0.6033 | 0.9813 | 0.1052 |
|--------|--------|--------|--------|--------|--------|
| 0.1816 | 0.7484 | 0.1699 | 0.7350 | 0.6430 | 0.8803 |

**2C.** Prove that P(A|B) = P(A) implies P(B|A) = P(B), provided that  $P(A) \neq 0$  and  $P(B) \neq 0$ .

[5+3+2]

**3A.** You are required to find the average annual salary of a software engineer in India. Suppose you take a random sample and find that mean monthly salary is Rs.53,145/- and you estimated the standard deviation equal to Rs.16, 451/-. Find a 95% confidence interval for the mean salary, given that your sample had size

(a) n = 36, (b) n = 64, (c) n = 144 (Take  $Z_{0.025} = 1.96$ ).

(d) Does the margin of error increase or decrease as n gets larger. What effect did sample size n have on the margin of error?

(e) Does the confidence interval get longer or shorter as the sample size increases? Is that what you expect?

- **3B.** A consulting firm rents cars from three agencies, 20% from agency D, 20% from agency E and 60% from agency F. If 10% of the cars from D, 12% of the cars from E and 4% cars from F have bad tires, what is the probability that the firm will get a car with bad tires?
- **3C.** Out of Chernoff, Markov and chebyshev's in equality, which one can be used for negative random variables? Justify your answer.

[5+3+2]

4A. A juice center in the MIT college campus near KC has been having average sales of 500 glasses per day. Due to the construction of new hostel block, increase in sales is expected. During the first 12 days after the start of the new hostel, the daily sales were as under:

550, 570, 490, 615, 505, 580, 570, 460, 600, 580, 530, 526

On the basis of this sample information, can one conclude that juice center sales have increased significantly due to the new hostel built? Use 5 per cent level of significance.

- 4B. Find random variate generator for exponential distribution
- 4C. If three random variables have the joint density function

$$\begin{split} f(x, y, z) &= k \; (x + y) \; e^{-z} & \text{for } 0 < x < 1, \, 0 < y < 2, \, z > 0 \quad \text{and} \\ f(x, y, z) &= 0, & \text{Otherwise.} \end{split}$$

Find the value of k.

[5+3+2]

**5A.** The following are data on the ages and income of 10 executives working for the same company and the number of years they went to college.

| Age            | Years   | Income (Y)        |
|----------------|---------|-------------------|
| $\mathbf{X}_1$ | College | (in Rs. 10,000/-) |
|                | $X_2$   |                   |
| 37             | 4       | 81                |
| 27             | 4       | 71                |
| 45             | 0       | 85                |
| 35             | 2       | 77                |
| 32             | 1       | 73                |
| 27             | 3       | 70                |

| 38 | 4 | 82 |
|----|---|----|
| 42 | 2 | 84 |
| 30 | 1 | 71 |
| 50 | 0 | 90 |

Fit an equation  $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$  to the given data, and use it to estimate how much on the average an executive working for this company will make if he is 40 years old and had 4 years of college.

- **5B.** Using Chernoff bound, upper bound the probability of getting less than n/2 heads in n tosses of a fair coin. Using the above bound, also calculate the probability of getting less than 10 heads in 100 tosses.
- **5C.** Show that as n tending to infinity binomial distribution with mean  $np = \lambda$  tends to Poisson distribution.

## [5 + 3 + 2]

6A. Buses arrive at Manipal bus stand according to a Poisson process with a mean of one bus per 10 minutes. Generate a random variate, N, which represents the number of arriving buses during a 1 hour time slot. Also calculate expected number of random numbers required to find 1000 Poisson variates. Make use of following random numbers to generate the Poisson variates:

0.43570.41410.83530.99520.80040.79450.15300.05120.01010.0019

- **6B.** A Company insures homes in three cities, J, K, L. The losses occurring in these cities are independent. The moment-generating functions for the loss distributions of the cities are  $M_J$  (t) =  $(1 2t)^{-3}$ ,  $M_K(t) = (1 2t)^{-2.5}$ ,  $M_L(t) = (1 2t)^{-4.5}$ . Let X represent the combined losses from the three cities. Calculate Expectation and variation of X.
- **6C.** Suppose the mean life span of English Springer Spaniel dogs is normally distributed with a mean of 13 years and a standard deviation of 1.5 years. Given that z-table value for Z = 0.67 and Z = 3.33 are 0.7486 and 0.9996 respectively. Using Central Limit Theorem find the following:

(a) What is the probability that a randomly selected English springer Spaniels will live to be 14 years or older?

(b) What is the probability that a randomly selected sample of 25 English Springer Spaniels will have a mean life span of 14 years or more?

[5 + 3 + 2]