Reg.No.									

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

I SEMESTER B.S. DEGREE EXAMINATION – NOV. / DEC. 2016

SUBJECT: MATHEMATICS - I (MA 111)

(COMMON TO ALL BRANCH)

Tuesday, 6 December 2016

Time: 3 Hours **Max. Marks: 100**

- ✓ **Answer ANY FIVE full Questions.**
- ✓ **Missing data, if any, may be suitably assumed**

1A. If $y=\left(x+\sqrt{x^2+1}\right)^m$, then prove that $\left(1+x^2\right)y_{n+2}+(2n+1)xy_{n+1}+\left(n^2-m^2\right)y_n=0.$ Hence determine $y_n(0)$.

1B. Find the area bounded by the parabolas $y^2 = 5x + 6$ and $x^2 = y$.

1C. Obtain a reduction formula for $\int \sin^n x dx$ when n is a non-negative integer and evaluate $\int_0^{\frac{\pi}{2}} \cos^4 x dx$. **(7 + 7 + 6)**

2A. Show that the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(ax)^{\frac{2}{3}}+(by)^{\frac{2}{3}}=\left(a^2-b^2\right)^{\frac{2}{3}}$.

2B. Find the area bounded by the curve $r^2 = a^2 \cos 2\theta$.

2C. Evaluate: i) $\int_0^{\pi} \frac{\sqrt{1-\cos x}}{1+\cos x} \sin^2 x\, dx$ ii) $\int_0^{\infty} \frac{x^2\ dx}{(1+x^2)^{\frac{7}{2}}}$.

(7 + 7 + 6)

3A. If 0 < a < b < 1, then prove that $\frac{b-a}{\sqrt{1-a^2}}<\sin^{-1}b-\sin^{-1}a<\frac{b-a}{\sqrt{1-b^2}}$.

Hence deduce that $\frac{\pi}{6}-\frac{1}{2\sqrt{3}}<\sin^{-1}\frac{1}{4}<\frac{\pi}{6}-\frac{1}{\sqrt{15}}$.

3B. The area enclosed by astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is revolved about x-axis. Find the volume of the solid generated.

3C. Trace the curve $y^2(a^2+x^2)=x^2(a^2-x^2)$ with explanation.

(7 + 7 + 6)

4A. Expand $\tan^{-1}x$ in powers of $(x-1)$ up to term containing $(x-1)^5$.

4B. Find the length of an arch of cycloid $x = a(\theta + sin\theta), y = a(1 + cos\theta)$.

4C. Trace the curve $x = \mathrm{a}\cos^3\theta,\ y = b\, sin^3\theta, a > b$ with explanation.

(7 + 7 + 6)

5A. Find the angle between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$.

5B. Find the equation of the plane that passes through the point (-1, 1, -4) and perpendicular to each of the planes -2x +y + z + 2 = 0 and x + y – 3z +1 = 0.

5C. Trace the curve $r = a(\,1 + sin\theta), a > 0$ with explanation.

(7 + 7 + 6)

6A. Show that the radius of curvature at any point θ on the cycloid $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$ is $4a\cos\frac{\theta}{2}$.

6B. Find the equations of two planes which bisects the angle between the planes 3x – 4y + 5z = 3, 5x + 3y – 4z =9. Specify the one which bisects the acute angle.

6C. State the values of x for which the following series converge

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots \infty.$ **(7 + 7 + 6)**

7A. Find the n^th derivative of the following functions:

(i) cosh4xcos3x (ii) $\log\sqrt{\frac{2x+1}{x-2}}$

7B. Find the shortest distance between the straight lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.

7C. Evaluate: (i) $\lim_{x \to o} \frac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}$ (ii) $\lim_{x \to o} \frac{(1+x)^{\frac{1}{x}} - e}{x}$.

(7 + 7 + 6)

8A. State the Cauchy's mean value theorem and verify for the functions $\frac{1}{x^2}$ and $\frac{1}{x}$ in (a, b), 0 < a < b.

8B. Find the equation of the sphere having its center on the plane 4x – 5y – z = 3 and passing through the circle $x^2 + y^2 + z^2 - 2x - 3y + 4z + 8 = 0,\ x - 2y + z = 8$.

8C. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots \infty$.

(7 + 7 + 6)