Reg. No.										

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

II SEMESTER B.S. DEGREE EXAMINATION – NOV. / DEC.2016

SUBJECT: MATHEMATICS -II (MA 121)

(BRANCH: COMMEN TO ALL)

Monday, 5 December 2016

Time: 3 Hours **Max. Marks: 100**

- **Answer ANY FIVE full Questions.**
- **Draw diagrams and equations whenever necessary.**

1A. If $u = \operatorname{cosec}^{-1}\left[\dfrac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]^{1/2}$, prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \dfrac{\tan u}{12}\left(\dfrac{13}{12} + \dfrac{\tan^2 u}{12}\right)$.

1B. Evaluate $\int_0^1 \int_0^{1-x} e^{y/(x+y)}\, dxdy,$ using the transformation $x + y = u,\ y = uv.$

1C. Prove that any orthogonal set of nonzero vectors is linearly independent.

(8+8+4)

2A. Verify Green's theorem for $\int_C \left(3x - 8y^2\right)dx + \left(4y - 6xy\right)dy$, where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1.$

2B. (i). Find the area common to the circles $r = a$ and $r = 2a\cos\theta$ using double integrals.

(ii). Changing to polar coordinates, evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\, dxdy$.

2C. Evaluate $\int_0^\infty \sqrt{x}\, e^{-x^3} dx$.

(8+8+4)

3A. (i). If $x^x y^y z^z = c,$ show that at $x = y = z,$ $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{-1}{x\log(ex)}$.

(ii). If $H = f(y - z,\ z - x,\ x - y),$ prove that $\dfrac{\partial H}{\partial x} + \dfrac{\partial H}{\partial y} + \dfrac{\partial H}{\partial z} = 0.$

3B. Prove that $\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m).$

3C. Using Gauss Jordan method, find the inverse of the matrix $A=\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

(8+8+4)

4A. (i). Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$.

(ii). Find the directional derivative of $\phi=x^2+y^2+xz$ at the point $(2,-1,3)$ in the direction of $A=i+2j+k.$

4B. Evaluate $\int_0^{\infty} dx \int_0^{\infty} dy \int_0^{\infty} \frac{dz}{\left(1+x^2+y^2+z^2\right)^2}$ using spherical polar coordinates.

4C. If the kinetic energy T is given by $T=\frac{1}{2}mv^2$, find approximately the change in T as m changes from 49 to 49.5 and v from 1600 to 1590.

(8+8+4)

5A. (i). Using divergence theorem evaluate $\iint_S \vec{A}\cdot n\, dS$, where $\vec{A}=4xi-2y^2j+z^2k$ and S is the surface bounded by $x^2+y^2=4,\ z=0$ and $z=3.$

(ii). Show that $\vec{F}=(6xy+z^3)i+(3x^2-z)j+(3xz^2-y)k$ is irrotational.

5B. (i). Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}}\,d\theta=\pi$.

(ii). Evaluate: $\int_0^1 \left(x\log x\right)^4 dx$

5C. At the distance of 50m from the foot of the tower of elevation at the top is 30^0. If the possible error in measuring the distance and elevation are 2cm and 0.05^0. Find approximate error in calculating height.

(8+8+4)

6A. Show that the rectangular solid of maximum volume can be inscribed in a sphere is a cube.

6B. Let $S = \{a_1, a_2, a_3\}$ be a basis for R^3 where $a_1 = (1,1,1),\ a_1 = (-1,0,-1)$, $a_1 = (-1,2,3)$. Use Gram Schmidt process to transform S to an orthonormal basis of R^3.

6C. Evaluate $\int\int y\ dxdy$ over the area bounded by $y = x^2$ and $x + y = 2$.

(8+8+4)

7A. Find the maximum and minimum values of $f(x,\ y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

7B. Test for consistency and solve the following system of equations by Gauss-Elimination method.

$$3x + 3y + 2z = 1$$

$$x + 2y = 4$$

$$10y + 3z = -2$$

$$2x - 3y - z = 5$$

7C. Find the work done in moving a particle in the force field $\vec{F} = 3x^2 i + (2xz - y)j + zk$, along the straight line from $(0,0,0)$ to $(2,1,3)$.

(8+8+4)

8A. The temperature T at any point $(x,\ y,\ z)$ in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

8B. Find the volume inside the cone $x^2 + y^2 = z^2$ bounded by the sphere $x^2 + y^2 + z^2 = a^2$.

8C. If $v = r^m$ where $r^2 = x^2 + y^2 + z^2$, show that $v_{xx} + v_{yy} + v_{zz} = m(m+1)r^{m-2}$.

(8+8+4)