Reg. No.										

INTERNATIONAL CENTRE FOR APPLIED SCIENCES

(Manipal University)

III SEMESTER B.S. DEGREE EXAMINATION - NOV. / DEC. 2016

SUBJECT: MATHEMATICS -III (MA 231)

(BRANCH: COMMON TO ALL)

Monday, 21 Nov. 2016

Time: 3 Hours **Max. Marks: 100**

- **Answer ANY FIVE full Questions.**
- **Draw diagrams and equations whenever necessary.**

1A. i) Show that $\int_0^\infty t^3 e^{-t} sintdt = 0$, ii) Find $L\left[e^{-t}\int_0^t \frac{sint}{t}dt\right]$.

1B. Solve $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$.

1C. Form differential equation of all circles with center at (h,k) and radius a.

[8+8+4]

2A. Solve $y^{\prime v} + 2y'' + y = x^2 cosx$.

2B. Using the method of separation of variables, solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

2C. If $w = \emptyset + i\varphi$ represents the complex potential for an electric field and $\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$ determine the function $\emptyset$. **[8+8+4]**

3A. Find by Taylor's series method the value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x,\ y(0) = 0$. Compare the solution obtained with the exact solution.

3B. Show that polar form of Cauchy-Riemann equations are

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Hence deduce $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

3C. Solve $y'' - 4y = \cosh(2x-1)$. **[8+8+4]**

4A. Express the following function in terms of unit step function and hence find it's Laplace transform.

$$f(t) = \begin{cases} 0\ ; & 0 < t < 1 \\ t-1\ ; & 1 < t < 2 \\ 1\ ; & t > 2 \end{cases}$$

4B. Verify Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i, -1+i$ and $-1-i$.

4C. Using Modified Euler's method, find an approximate value of y when $x = 0.2$, given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$. **[8+8+4]**

5A. Solve $\frac{dx}{dt} - 7x + y = 0\,; \frac{dy}{dt} - 2x - 5y = 0.$

5B. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$ and $x = 0.4$

5C. Find $L[(1 - e^{2t})u(t-1)]$ **[8+8+4]**

6A. Solve $x^2y'' + xy' + 9y = 3x^2 + \sin(3logx)$.

6B. Solve by the method of transforms, the equation $y''' + 2y'' - y' - 2y = 0$, given that $y(0) = y'(0) = 0$ and $y''(0) = 6$

6C. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0.$ **[8+8+4]**

7A. Solve $y'' + y = \frac{1}{1+sinx}$ using the method of variation of parameters.

7B. Define Periodic function. Find the Laplace transform of the function

$$f(t) = \begin{cases} sinwt\,; 0 < t < \frac{\pi}{w} \\ 0 \quad ;\ \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases}$$

7C. Find the sum of the residues of $f(z) = \frac{sinz}{z\,cosz}$ at it's poles inside the circle $|z| = 2$.

[8+8+4]

8A. Solve $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2ydy = 0.$

8B. i) Find $L^{-1}\left[\frac{1}{s(s^2+a^2)}\right]$, ii) Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$

8C. Form the partial differential equations by eliminating arbitrary functions from $z = (x + y)\varphi(x^2 - y^2)$ **[8+8+4]**