Reg. No. ☐☐☐☐☐☐☐☐☐☐

MANIPAL INSTITUTE OF TECHNOLOGY

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A Constituent Institution of Manipal University

INSPIRED BY LIFE

III SEMESTER B.TECH. (Mech/ IP/Auto/Aero/MT)

END SEMESTER EXAMINATION, NOV/DEC 2016

SUBJECT: ENGINEERING MATHEMATICS III MAT 2101

23/11/2016

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

<table>
<tr><td>1A.</td><td>Solve $y'' + xy = 1$ Subjected to the conditions $y(0) = 0, y'(1) = 1$ by taking h = 0.5</td><td>04</td></tr>
<tr><td>1B.</td><td>Solve $x^2y'' + xy' + (x^2 - 3)y = 0$ Subjected to the conditions $y(1) = 0 , y(2) = 2$ by taking h = 0.25.</td><td>03</td></tr>
<tr><td>1C.</td><td>Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $,0 < x < 1,\ t > 0$ subjected to the conditions $u(x,0) = 100(x - x^2), u(0,\ t) = u(1,t) = 0$. Compute u for one time step with h = 0.25 using Crank Nicolson’s method</td><td>03</td></tr>
<tr><td>2A.</td><td>Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ $0 < x < 1 , t > 0$ with $u(x,0) = 100(x - x^2) , \frac{\partial u}{\partial t}(x,0) = 0 ,\ u(0,t) = u(1,t) = 0$ Choosing h = 0.25 for four time steps</td><td>04</td></tr>
<tr><td>2B.</td><td>Obtain the Fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi} ; & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} ; & 0 \leq x \leq \pi \end{cases}$, Given that $f(x + 2\pi) = f(x)$.</td><td>03</td></tr>
<tr><td>2C.</td><td>Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the below table.
<table>
<tr><td>x^o</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td></tr>
<tr><td>y</td><td>4</td><td>8</td><td>15</td><td>7</td><td>6</td><td>2</td></tr>
</table></td><td>03</td></tr>
<tr><td>3A.</td><td>Determine whether $\vec{F} = (x + 2y + 4z\ )\hat{\imath} + (2x - 3y - z)\hat{\jmath} + (4x - y + 2z)\hat{k}$ is conservative? If so find scalar potential.</td><td>04</td></tr>
</table>

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3B.	From the Fourier integral show that $\int_0^{\infty} \frac{s \sin sx}{1+s^2}\, ds \;=\; \frac{\pi}{2} e^{-x} \;\; (x > 0)$	**03**
3C.	Find the Fourier transform of $f(x) = \begin{cases} 1 - \lvert x\rvert \,, & \text{for } \lvert x\rvert < 1 \\ 0, & for\ \lvert x\rvert < 1 \end{cases}$ and deduce that $\int_0^{\infty} \left(\frac{sint}{t}\right)^4 dt = \frac{\pi}{3}$.	**03**
4A.	Derive the one dimensional wave equation with suitable assumptions.	**04**
4B.	Use Gauss divergence theorem to evaluate $\iint_S \vec{f}.\,n\, ds$, where $\vec{f}$ = 4x i - 2y^{2}j + z^{2}k and S is the surface bounding the region $x^2 + y^2 = 4,\ z = 0$ and $z = 3$.	**03**
4C.	Given $\vec{A} \; = (\, yz + 2x\,)\hat{\imath} + xz\hat{\jmath} + (\, xy + 2z)\hat{k}$. Evaluate $\int \vec{A}.\,d\vec{r}$ along the curve $x^2 + y^2 \; = 1$ & $z = 1$ in the positive direction of $(0, 1, 1)$ to $(1, 0, 1)$	**03**
5A.	Solve the partial differential equation $\mathrm{U_{xx}} + 2\mathrm{U_{xy}} + \mathrm{U_{yy}} = 0$ using the transformation $v = x, z = x - y$	**04**
5B.	Verify Green's theorem for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ Where C is the boundary of the region bounded by $x = 0, y = 0\ and\ x + y = 1.$	**03**
5C.	Let $\boldsymbol{A} = A_1\boldsymbol{i} + A_2\boldsymbol{j} + A_3\boldsymbol{k}$ and φ is a scalar function then prove that $\boldsymbol{\nabla}\,.\,(\varphi\boldsymbol{A}) = (\boldsymbol{\nabla}\varphi)\cdot\boldsymbol{A} + \varphi(\boldsymbol{\nabla}\cdot\boldsymbol{A})$. Hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$.	**03**