

# MANIPAL INSTITUTE OF TECHNOLOGY

A Constituent Institution of Manipal University

## **III SEMESTER B.TECH. (CHEMICAL/BIOTECH)**

### END SEMESTER EXAMINATIONS, NOV/DEC 2016

## SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2103]

#### REVISED CREDIT SYSTEM (28/11/2016)

Time: 3 Hours

MAX MARKS: 50

#### Instructions to Candidates:

- ✤ Answer ALL the questions.
- ✤ Missing data may be suitable assumed.

| 1A. | Find the Fourier series expansion of the function<br>$f(x) = x - x^2$ in $(-\pi, \pi)$ ; $f(x + 2\pi) = f(x) \forall x$ .<br>Hence show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ | 4 |
|-----|--|---|
| 1B. | Find the equation of the tangent plane and the normal line to the surface $x^2yz - 4xyz^2 = -6$ at the point (1, 2, 1).  | 3 |
| 1C  | Form the PDE by eliminating the arbitrary constants 'a' and 'b' $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$ , where $\propto$ is a parameter.  | 3 |
| 2A. | Verify Green's theorem in the plane<br>for $\oint_C (x^2 - xy^3) dx + (y^2 - 2xy) dy$ , where C is a square with vertices<br>(0,0)(2,0), (0,2) and (2,2).  | 4 |
| 2B. | Obtain the half range cosine series expansion of the function $f(x) = x \sin x$<br>in $(0, \pi)$ .   | 3 |
| 2C. | Find the analytic function $f = u + iv$ , where $v = \log(x^2 + y^2) + x - 2y$   | 3 |

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| MANIPAL INSTITUTE OF TECHNOLOGY |  |  |                    |                        |                    |      |       |      |      |      |      |   |   |
| 3A.                             | Find the residues of the following functions at their singularities.<br>i. $f(z) = \frac{1}{z^3(z+4)}$<br>ii. $f(z) = \frac{1}{e^{2z}z^2}$         |  |                    |                        |                    |      |       |      |      |      | 4    |   |   |
| 3B.                             | Find the Fourier transform of the function<br>$f(x) = \begin{cases} a -  x  &  x  < a \\ 0 &  x  > a \end{cases}$                                  |  |                    |                        |                    |      |       |      |      |      |      | 3 |   |
| 3C.                             | Prove that $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is conservative. Find its scalar potential.                      |  |                    |                        |                    |      |       |      |      |      |      | 3 |   |
| <b>4</b> A.                     | Derive D'Alembert's solution of wave equation.   |  |                    |                        |                    |      |       |      |      |      |      | 4 |   |
| 4B.                             | Solve $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0$ using the method of separation of variables.                     |  |                    |                        |                    |      |       |      |      |      |      | 3 |   |
| 4C.                             | If $f(\alpha) = \int_{C} \frac{3z^2 + 7z + 1}{z - \alpha} dz$ , where $C:  z  = 4$ . Evaluate<br>i. $f(3)$<br>ii. $f'(1 - i)$<br>iii. $f''(1 - i)$ |  |                    |                        |                    |      |       |      |      |      |      | 3 |   |
| 5A.                             | Find all the possible expan  | sions of $f(z)$                                    | $) = -\frac{1}{z}$ | $\frac{1}{2-5}$        | <i>z</i> + 6       | - wi | ith c | ente | er z | = 1. |      |   | 4 |
| 5B.                             | Find $F_c\{e^{ax}\}$ and hence evaluate<br>$F_c\{\frac{1}{1+x^2}\}$ and $F_s\{\frac{x}{1+x^2}\}$   |  |                    |                        |                    |      |       |      |      |      |      | 3 |   |
| 5C.                             | Use Divergence theorem f<br>in the first octant bounded  | for $\vec{A} = 2x^2y\hat{i}$<br>by $y^2 + z^2 = 9$ | $-y^2$ and         | $\hat{j} + 4$<br>l x = | $xz^2\hat{k}$ = 2. | tal  | ken   | ove  | r th | e re | gior | n | 3 |