

Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)

III SEMESTER B.TECH (CS/ICT/CC- ENGINEERING)

END SEMESTER EXAMINATION, NOVEMBER - DECEMBER 2016

SUBJECT: ENGINEERING MATHEMATICS III [MAT 2105]

REVISED CREDIT SYSTEM

Time: 3 Hours

MAX. MARKS: 50

Instructions to Candidates:

- ❖ Answer **ALL** the questions.
- ❖ All the questions carry (4+3+3) marks.

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| 1A. | Let $E(x_1, x_2, x_3) = \overline{(x_1 \wedge x_2)} \vee \overline{(x_2 \wedge x_3)}$ be a Boolean expression over the two-valued Boolean algebra. Write $E(x_1, x_2, x_3)$ in both DNF and CNF. | 4 Marks |
| 1B. | Let (A, \leq) be a distributive lattice. Show that, if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some $a \in A$, then $x = y$. | 3 Marks |
| 1C. | Show that a lattice (A, \leq) is distributive if and only if for any element a, b, c in A , $(a \vee b) \wedge c \leq a \vee (b \wedge c)$. | 3 Marks |
| 2A. | Let P_n be the unrestricted partitions of n , and P_n^* be the number of partitions of n without unit parts. Using generating function or otherwise show that, for $n > 1$, $P_n^* = P_n - P_{n-1}$. Generalize this result to find the formula for the number of partitions of n without part of size k . | 4 Marks |
| 2B. | Show that the proportion of permutations of symbols $\{1, 2, 3, \dots, n\}$ which does not contain i in the i^{th} place is approximately $\frac{1}{e}$. | 3 Marks |
| 2C. | For $n = 5$ and marks 1, 2, 3, 4, 5 with initial permutation 12345, obtain the 43 rd and 107 th permutations in a) Lexicographical order b) Fike's order. | 3 Marks |
| 3A. | Given distance matrix of the network, using Dijkstra's algorithm, find the | 4 Marks |

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| | shortest weighted path from C to all other vertices. | |
| | $\text{Distance matrix} = \begin{matrix} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 0 & 5 & 6 & \infty & 17 & \infty \\ 3 & 0 & 4 & \infty & \infty & 7 \\ \infty & \infty & 0 & 6 & \infty & 11 \\ 11 & \infty & 7 & 0 & 9 & 4 \\ \infty & \infty & \infty & \infty & 0 & 5 \\ 11 & \infty & \infty & 9 & 4 & 0 \end{bmatrix} \end{matrix}.$ | |
| 3B. | Prove that a graph is bipartite if and only if all its cycles are even. | 3 Marks |
| 3C. | Show that a (p, q) – graph G is a tree if and only if it is connected and $p = q + 1$. | 3 Marks |
| 4A. | Show that the following premises are inconsistent. (i). If Jack misses many classes through illness, then he fails high school. (ii). If Jack fails high school, then he is uneducated. (iii). If Jack reads a lot of books, then he is not uneducated. (iv). Jack misses many classes through illness and reads a lot of books. | 4 Marks |
| 4B. | Show that subgroup of a cyclic group is again cyclic. | 3 Marks |
| 4C. | Show that any group with at most five elements is abelian. | 3 Marks |
| 5A. | Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$ and with justification show that the converse is not true. | 4 Marks |
| 5B. | Show that number of partitions of n with at most k part is same as the number of partitions of n with no part greater than k . Hence get an expression for number of partitions of n with exactly k parts. | 3 Marks |
| 5C. | Let G be a group and H be subgroup of G . Then prove that any two right co-sets of H in G are either identical or disjoint. | 3 Marks |