

Reg. No.



# MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

## III SEMESTER B.TECH. (E&C/EE/ICE/BM ENGINEERING)

### END SEMESTER EXAMINATIONS, NOV. 2016

### SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2102]

#### REVISED CREDIT SYSTEM (25/11/2016)

Time: 3 Hours

MAX. MARKS: 50

#### Instructions to Candidates:

❖ Answer **ALL** the questions.

1A.	Find the half range Fourier cosine series expansion of $f(x) = 1 - \frac{x}{l}, 0 < x < l$ . Also draw the graph of corresponding periodic extension of $f(x)$ .	3
1B.	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, &  x  \leq 1 \\ 0, &  x  > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{\sin t - t \cos t}{t^3} dt$ .	3
1C.	Expand $f(x) = x \sin x, 0 \leq x \leq 2\pi, f(x+2\pi) = f(x)$ as a Fourier series and hence evaluate $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$ .	4
2A.	Find the Fourier cosine transform of $e^{-ax}, a > 0$ , and hence find $F_s \left\{ \frac{x}{a^2 + x^2} \right\}$	3
2B.	Find the analytic function $f(z) = u + iv$ for which $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$	3
2C.	(i) Find all possible expansion of $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ about $z = -1$ . (ii) Expand $f(z) = \sin 2x$ about $z = \pi/4$ .	4
3A.	If $f(z) = u + iv$ is analytic function of $z$ , show that $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u^p = p(p-1)u^{p-2}  f'(z) ^2$	3



<b>3B.</b>	Evaluate $\oint_C \frac{z^2}{(z+1)^2(z^2+4)} dz$ where $C:  z-2i =3$ .	<b>3</b>
<b>3C.</b>	Verify Green's theorem for $\oint_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$ Where C is the boundary of the region defined by $x=0$ , $y=0$ and $x+y=1$ .	<b>4</b>
<b>4A.</b>	Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$ .	<b>3</b>
<b>4B.</b>	Show that $\vec{F} = (y^2 \cos xy + z^3)\hat{i} + (\sin xy + xy \cos xy - 4z)\hat{j} + (3xz^2 - 4y)\hat{k}$ is conservative, find its scalar potential and work done in moving an object in this field from $(0, 1, -1)$ to $\left(\frac{\pi}{2}, -1, 2\right)$ .	<b>3</b>
<b>4C.</b>	If $f(r)$ is a differentiable function of $r =  \vec{r} $ then show that $f(r)\vec{r}$ is irrotational. Find $f(r)$ so that $f(r)\vec{r}$ is also solenoidal.	<b>4</b>
<b>5A.</b>	Evaluate $\oiint_S \vec{A} \cdot \hat{n} dS$ where $\vec{A} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ and $\hat{n}$ , positive unit normal to Surface S of the region in the first octant bounded by $y^2 + z^2 = 9$ and $x=2$ .	<b>3</b>
<b>5B.</b>	Assuming the most general solution, solve the one dimensional heat equation $u_t = c^2 u_{xx}$ in a laterally insulated bar of length L whose ends are kept at zero and the initial temperature is $f(x) = \begin{cases} x, & 0 < x < L/2 \\ L-x, & L/2 < x < L \end{cases}$	<b>3</b>
<b>5C.</b>	Solve the partial differential equation $u_{tt} = c^2 u_{xx}$ using the transformations $v = x + ct$ , $w = x - ct$ subject to the conditions $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$ .	<b>4</b>