Reg. No.										

MANIPAL UNIVERSITY

THIRD SEMESTER B.S. (ENGG.) DEGREE EXAMINATION – DECEMBER 2015

SUBJECT: MATHEMATICS – III (MA 231)

(BRANCH: CE/CS/E&C/E&E/BIOM/MECHANICAL/IP/IND.BIOTECH/CHEMICAL/CIVIL)

Monday, December 07, 2015

Time: 10:00-13:00 Hrs. Max. Marks: 100

✍ **Answer any FIVE full questions in each part.**

PART – A

1A. Solve $\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=0, x(0)=0, \frac{dx}{dt}(0)=1$.

1B. State the Cauchy- Riemann equations in the polar form.

1C. Form the partial differential equations (by eliminating arbitrary function) from $z = f\left(x^2-y^2\right)$.

1D. Obtain the differential equation of all circles of radius a and centre (h, k)

1E. Find the inverse Laplace transform of $\frac{s+3}{s^2-4s+13}$.

1F. Show that $\log\sqrt{x^2+y^2}$ is a harmonic function.

1G. Find the Laplace transform of $\sqrt{t}+\cos 2t$.

(2 marks × 5 = 10 marks)

PART – B

2A. Solve $\left(D^2-2D+1\right)y = e^x \log x$ using method of variation of parameters.

2B. Derive Cauchy Riemann equations in Cartesian form.

2C. Solve $\left(y^4+2y\right)dx+\left(xy^3-4x+2y^4\right)dy=0$

2D. Using convolution theorem, evaluate $L^{-1}\left(\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right)$

2E. Evaluate $\int\left(x^2+ixy\right)dz$ from $A(1,1)$ to $B(2,4)$ along the curve $x=t$ and $y=t^2$.

2F. Find $y(0.4)$ using Euler's method from $\frac{dy}{dx}=x+y$ with $y(0)=1$ by taking h = 0.1.

2G. Find $L^{-1}\left(\log\frac{s+1}{s-1}\right)$.

(4 marks × 5 = 20 marks)

PART – C

3A. If $f(z)$ is an analytic function then prove that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\left|f(z)\right|^2=4\left|f'(z)\right|^2$

3B. Evaluate i) $\int_0^\infty te^{-3t}\sin t\,dt$ ii) $L\left\{\frac{\cos at-\sin bt}{t}\right\}$

3C. Apply Runge-Kutta fourth order method to find an approximate value of y when x = 0.4 given that $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ and y(0) = 1 by taking h = 0.2.

3D. Solve $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial t}=u$ where $u(x,0)=6e^{-3x}$, using method of separation of variables.

3E. Solve the differential equation using Laplace transforms $y''(t)-3y'(t)+2y(t)=4t+e^{3t}$, $y(0)=y'(0)=-1$.

3F. Solve the simultaneous equations $\frac{dx}{dt}+2y+\sin t=0, \frac{dy}{dt}-2x-\cos t=0$ given that $x(0)=0, y(0)=1$.

3G. State Cauchy's integral formula and hence evaluate $\oint_C \frac{3z^2+z}{z^2-1}dz$ over $|z-1|=1$.

(6 marks × 5 = 30 marks)

PART – D

4A. Using Modified Euler's method, an approximate value of $y(0.4)$ with $h=0.2$ from $\frac{dy}{dx}=y+e^x, y(0)=0$.

4B. Solve $(D-2)^3 y=8\left(e^{2x}+\sin 2x+x^2\right)$

4C. State Residue theorem. Evaluate $\oint_c \frac{z-3}{z^2+2z+5}dz$ where C is the circle $(i)|z|=1$ $(ii)|z+-i|=2$ $(iii)|z+1+i|=2$.

4D. Solve $(3y+2x+4)dx-(4x+6y+5)dy=0$.

4E. By integrating around a unit circle, evaluate $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}d\theta$.

4F. Solve $u_{xx}+u_{xy}-2u_{yy}=0$ using the transformations $v=x+y, z=2x-y$.

4G. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\sin(\log x)$.

(8 marks × 5 = 40 marks)