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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

THIRD SEMESTER B.TECH DEGREE MAKEUP EXAMINATION- JAN. 2016
ENGINEERING MATHEMATICS III (MAT 2102)
(COMMAN TO E&C, E&E, ICE, Bio-Med. Engg)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max. Marks : 50**

Note : a). Answer all questions. b). All questions carry equal marks

1A. Expand f(x)= $x-x^2$, $-\pi \le x \le \pi$, $f(x+2\pi) = f(x)$ as a Fourier series and hence evaluate $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

1B. Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1,-2,1).

1C. If $f(z) = u + iv$ is an analytic function of $z = x + iy$, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

(4 + 3 + 3)

2A. Find the Fourier transform of $f(x) = e^{-ax}, a > 0$, hence evaluate

$$\int_0^{\infty} \frac{\cos xt}{a^2 + t^2} dt \text{ and } F\left\{xe^{-ax}\right\}.$$

2B. Solve $u_x + u_y = 2(x + y)u$ by separating the variables.

2C. Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$. (4 + 3 + 3)

3A. Find $F_c\left\{e^{-a^2x^2}\right\}$ and $F_s\left\{xe^{-a^2x^2}\right\}, 0 < a.$

3B. Find the harmonic conjugate of v= $e^x(y\cos y + x\sin y)$.

3C. Evaluate $\iint_S (\nabla \times \vec{A}).\hat{n}\, dS$ where $\vec{A} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = 16$ above the xy-plane.

(4 + 3 + 3)

4A. Obtain all possible power series expansions of the function

$$f(z) = \frac{1}{z^2+3z+2} \text{ about } z = 0..$$

4B. Assuming the most general solution, find the deflection u(x, t) in a rod of length π, which is perfectly insulated laterally with ends kept at 0^0 C initial heat is given by u(x,0) = ksin3x.

4C. Obtain the half-range Fourier Cosine series expansion of $f(x) = 1 - \frac{x}{l}$, $0 < x < l$. Also sketch the corresponding periodic extension of f(x).

(4 + 3 + 3)

5A. State and prove Green's theorem .

5B. Evaluate $\oint_{|z|=4} \frac{z+1}{z(z^2+5z+6)} dz$

5C. Derive the D'Alembert's solution of the one dimensional wave equation.

(4 + 3 + 3)

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