Reg.No.									

MANIPAL INSTITUTE OF TECHNOLOGY
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THIRD SEMESTER B.E DEGREE END SEMESTER EXAMINATION
ENGINEERING MATHEMATICS III (MAT 2102)
(COMMAN TO E&C, E&E, ICE, Bio-Med. Engg)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max. Marks : 50**

Note : a). Answer all questions. b). All questions carry equal marks

1A. Expand , $f(t)=\begin{cases} \pi t, & 0 < t \le 1 \\ \pi(2-t), & 1 < t < 2 \end{cases}$, f(t+2) = f(t) and hence evaluate

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

1B. Find the directional derivative of $\varphi = 4e^{2x-y+z}$ at the point (1,1,-1) in the direction towards the point (-3,5,6).

1C. If $f(z) = u + iv$ is an analytic function of $z = x + iy$, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2 |f(z)|^{p-2} |f'(z)|^2$$

(4 + 3 + 3)

2A. Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & |x| \le a \\ 0, & |x| > a \end{cases}$

and hence evaluate $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$

2B. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} = 0$ using the transformations $v = x + y, z = 2x - y$.

2C. Prove that $\vec{A} = r^n \vec{r}$ is irrotational. Find n when it is also solenoidal.

(4 + 3 + 3)

3A. Find $F_c\{e^{-ax}\}, F_s\{e^{-ax}\}, 0 < a < 1,$ *and hence evaluate* $F_c\{xe^{-ax}\}, F_s\{xe^{-ax}\}$.

3B. Find the analytic function f(z) whose imaginary part is $e^{-x}(y\cos y - x\sin y)$.

3C. Evaluate $\oiint_S \vec{A}.\hat{n}\, dS$ where $\vec{A} = (2x-y)\hat{i} - 2y\hat{j} - 4z\hat{k}$ and S is the surface of the region bounded by x = 0, y = 0, z = 0, z = 3and $x^2 + y^2 = 16$ lying in the first octant.

(4 + 3 + 3)

4A. Obtain all possible power series expansions of the function

$f(z) = \dfrac{1}{z(z-1)^2}$ about $z = -1$.

4B. Assuming the most general solution, find the deflection u(x, t) in a vibrating string of length π, which is fixed at end points, starts vibration with an initial deflection u(x,0) = k(sinx-sin2x) and zero initial velocity.

4C. Obtain the half-range Fourier Cosine series expansion of f(x) = x(*l*-x), 0 < x < *l*. Also sketch the corresponding periodic extension of f(x).

(4 + 3 + 3)

5A. Verify Green's theorem in the plane for $\oint_C \left(xy - x^2\right) dx + x^2 y dy$,

where C is the triangle with vertices at (0,0), (1,0) and (1,1).

5B. Evaluate $\oint_{|z|=3} \dfrac{e^z}{z^3(z^2 - 3z + 2)} dz$

5C. Assuming the most general solution, find the temperature u(x, t) in a rod of length 10 cms , which is perfectly insulated laterally, ends are kept at zero degree temperature and whose initial temperature is given by, u(x ,0) = x(10-x)

(4 + 3 + 3)

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