Reg. No.										

Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)

III SEMESTER B.TECH (CHEMICAL/BIOTECHNOLOGY)

END SEMESTER EXAMINATIONS, NOV/DEC 2015

SUBJECT: ENGG.MATHEMATICS III [MAT 2103]

REVISED CREDIT SYSTEM

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitable assumed.

1A.	Expand $f(x)=2x-x^2$ in $(0,\ 3)$ as Fourier series. Hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\dots\dots\dots\dots=\frac{\pi^2}{12}$.	**4**
1B.	Find the angle between the surfaces $z=\left(x-\frac{\sqrt{6}}{6}\right)^2+\left(y-\frac{\sqrt{6}}{6}\right)^2$ and $z=x^2+y^2$ at P$\left(\frac{\sqrt{6}}{12},\frac{\sqrt{6}}{12},\frac{1}{12}\right)$.	**3**
1C.	Solve $u_{xy}-u_{yy}=0$ using the transformation $v=x,\ z=x+y$.	**3**
2A.	State Green's theorem and apply the same to evaluate $\int_C(2x^2-y^2)dx+(x^2+y^2)dy$, where C is the boundary of the area enclosed by x-axis and the upper half of the circle $x^2+y^2=a^2$.	**4**
2B.	Sketch the even periodic extension of the function $f(t)=\begin{cases}e^{-t} & 0<t<1\\ 0 & 1<t<2\end{cases}$. Also obtain the half range cosine expansion.	**3**
2C.	Find the analytic function $f=u+iv$, where $v=\log(x^2+y^2)+x-2y$	**3**
3A.	Find the residue of the following functions at their singularities: (i) $\frac{e^z}{(z-1)^3}$ (ii) $\frac{1}{1-\cos z}$	**4**

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3B.	Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & \lvert x\rvert < a \\ 0 & \lvert x\rvert > a \end{cases}$. Hence show that $\int_0^\infty \frac{sint - tcost}{t^3} dt = \frac{\pi}{4}$.	**3**
3C.	Solve by the method of separation of variables: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u;\ u(x,0) = 6e^{-3x}$	**3**
4A.	Derive the one dimensional wave equation by stating the appropriate physical assumptions.	**4**
4B.	Prove $\boldsymbol{F} = (y^2 cos\,x + z^3)\,\boldsymbol{i} + (2y\,sin\,x - 4)\,\boldsymbol{j} + (3xz^2 + 2)\boldsymbol{k}$ is a conservative force field. Find the scalar potential for **F.**	**3**
4C.	Show that *v(x,y)* = *-sinx sinhy* is harmonic. Find the conjugate harmonic of v.	**3**
5A.	If $f(\xi) = \int_C \frac{4z^2 + z + 5}{z - \xi} dz$, where C is the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$, find the value of $f(3.5), f(i), f'(-1)\ and\ f''(-i)$.	**4**
5B.	Prove the property $F_s\{xf(x)\} = -\frac{d}{ds}F_c$. Also find $F_c\left\{\frac{1}{1+x^2}\right\}$ and use the given property to find $F_s\left\{\frac{x}{1+x^2}\right\}$.	**3**
5C.	Let $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$. Evaluate $\iint_S F.nds$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	**3**