



Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)



III SEMESTER B.TECH (MECH/IP/MT/AUTO/AERO) END SEMESTER MAKEUP EXAMINATIONS, DEC 2015

SUBJECT: ENGG. MATHEMATICS III [MAT 2101]

REVISED CREDIT SYSTEM

Time: 3 Hours

MAX. MARKS: 50

Instructions to Candidates:

- ❖ Answer **ALL** the questions.
- ❖ Missing data may be suitable assumed.

1A.	Solve $(x^3 + 1)y'' + x^2y' - 4xy = 2$, $y(0) = 0$, $y(2) = 4$ with $h = 0.5$.	04
1B.	Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $0 < x < 1$, $0 < y < 1$ subjected to the conditions $u(x, 1) = u(0, y) = 0$, $u(1, y) = 9(y - y^2)$, $u(x, 0) = 9(x - x^2)$ by taking $h = \frac{1}{3}$	03
1C.	Show that $\vec{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is conservative. Find the work done in moving an object from $(1, -2, 1)$ to $(3, 1, 4)$ in this field.	03
2A.	Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$ with $u(x, 0) = 0$, $\frac{\partial u}{\partial t}(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 100 \sin \pi t$. Compute u for four time steps with $h = 0.5$.	04
2B.	Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -81xy$, $0 < x < 1$, $0 < y < 1$, $u(0, y) = 0$, $u(x, 0) = 0$, $u(1, y) = u(x, 1) = 100$ with $h = \frac{1}{3}$.	03
2C.	Obtain the Fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi}; & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}; & 0 \leq x \leq \pi \end{cases}$	03
3A.	Verify Green's theorem in the plane for $\oint (xy + y^2)dx + x^2 dy$, where the closed curve of the region is bounded by $y = x$ and $y = x^2$.	04



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3B.	Find a unit normal to the surface $-x^2yz^2 + 2xy^3z = 1$ at the point $P(1, 1, 1)$.	03														
3C.	Find the half range sine series for $f(x) = x$ in $(0, 2)$	03														
4A.	Find the directional derivative for the scalar function $\phi(x, y, z) = x^2yz + 4xz^2$ at $p(1, -2, -1)$ In the direction of $2i - j - 2k$.	04														
4B.	From the Fourier integral representation show that $\int_0^\infty \frac{s \sin sx}{1 + s^2} dx = \frac{\pi}{2} e^{-x} (x > 0)$	03														
4C.	Find the Fourier transform of $f(x) = \begin{cases} 1 - x & x < 1 \\ 0 & x > 1 \end{cases}$ and hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$.	03														
5A.	Derive one dimensional heat equation with necessary assumptions.	04														
5B.	Solve the partial differential equation $U_{xx} - 4U_{xy} + 3U_{yy} = 0$ using the transformation $v = x + y, \quad z = 3x + y$.	03														
5C.	Obtain the constant term and the coefficient of the first order sine and cosine term in the Fourier series expansion of <table border="1"><tr><td>x (in degree)</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td></tr><tr><td>y</td><td>9</td><td>18</td><td>24</td><td>28</td><td>26</td><td>20</td></tr></table>	x (in degree)	0	60	120	180	240	300	y	9	18	24	28	26	20	03
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