Reg.No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

THIRD SEMESTER B.Tech DEGREE END SEMESTER EXAMINATION- NOV 2015

SUB: ENGG. MATHEMATICS III (MAT 2101)(MECH/IP/MT/AUTO/AERO)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max. Marks : 50**

Note : a). Answer any FIVE full questions. b). All questions carry equal marks

1A. Solve $x^2y'' + xy' + (x^2 - 3)y = 0$, $y(1)=0$, $y(2)=2$ with $h = 0.25$.

1B. Solve $16\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$ subjected to the conditions $u(x,0) = 0 = u(0, t)$, $u(1,t) = 100t$, Compute u for two time steps with h = 0.25 using Crank Nicolson's method.

1C. Evaluate $\int_C \vec{F}.dr$ Where $\vec{F} = (x^2 + y^2)\, i - 2xy\, j$. C is the rectangle in xy plane bounded by y = 0, y = b, x = 0, x = a.

(4 + 3 + 3)

2A. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ $0 < x < 2$, $t > 0$ with $u(x,0) = 0$, $\frac{\partial u}{\partial t}(x,0) = 100(2x - x^2)$, $u(0,t) = u(2,t) = 0$ Choosing h = 0.5 for four time steps.

2B. With h=1, solve the Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10), \quad 0 < x < 3, \quad 0 < y < 3.\ u = 0 \text{ on the boundary}$$

2C. Obtain the Fourier series expansion of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

Hence deduce $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

(4 + 3 + 3)

3A. State prove the Green's theorem in the plane.

3B Find the values of the constants a, b, c such that directional derivative of $\emptyset = axy^2 + byz + cz^2x^3$ at the point (1, 2, –1) has the maximum magnitude of 64 in the direction parallel to z axis

3C. Find the half range sine series for $f(x) = \begin{cases} \frac{1}{4} - x\,; & 0 \le x < \frac{1}{2} \\ x - \frac{3}{4}\,; & \frac{1}{2} < x \le 1 \end{cases}$

(4 + 3 + 3)

4A. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ *and* $z = x^2 + y^2 = 3$ at the point (2, –1, 2)

4B. From the Fourier integral show that $\int_0^\infty \frac{\sin \pi s \sin s\theta}{1 - s^2}\, ds = \begin{cases} \frac{\pi}{2} \sin\theta\,, 0 \le \theta \le \pi \\ 0\,, \ \theta > \pi \end{cases}$

4C. Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$ and deduce that $F\left(e^{-\frac{x^2}{2}}\right) = e^{-\frac{s^2}{2}}$.

(4 + 3 + 3)

5A. Derive one dimensional wave equation with necessary assumptions.

5B. Solve the partial differential equation $U_{xx} - 4U_{xy} + 3U_{yy} = 0$ using the transformation v = x + y, z = 3x + y.

5C. Obtain the first three coefficients in the fourier cosine series for y, where y is given in the following table.

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(4 + 3 + 3)

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