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MANIPAL UNIVERSITY, MANIPAL

FIRST SEMESTER M.SC (APPLIED MATHEMATICS & COMPUTING) END SEMESTER EXAMINATION, NOVEMBER, 2015

SUB: DIFFERENTIAL EQUATIONS(MAT - 601)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

Note : a) Answer any FIVE full questions. b) All questions carry equal marks (3+ 3+ 4).

1A. If $y = x$ is one of the solution of the differential equation $(x^2+1)\,y'' - 2\,x\,y' + 2\,y = 0$. Find the general solution of $(x^2+1)\,y'' - 2\,x\,y' + 2\,y = 6(x^2+1)^2$.

1B. Obtain the Rodrigue's formula $P_n(x) = \dfrac{1}{n!\,2^n}\dfrac{d^n}{dx^n}(x^2-1)^n$ where $P_n(x)$ is Legendre polynomial of degree n.

1C. State and prove uniqueness theorem for Let $L(y) = y^{(n)} + a_1\,y^{(n-1)} + a_2 y^{(n-2)} + \cdots\cdots + a_n\,y = 0$ on an interval I containing a point x_0. With usual initial conditions

2A. Find a function $\varphi(x)$ which has a continuous derivative on $0 \le x \le 2$ which satisfies $\varphi(0) = 0$, $\varphi'(0) = 1$ and $y'' - y = 0$ for $0 \le x \le 1$ and $y'' - 9\,y = 0$ for $1 \le x \le 2$.

2B Prove that (i) $J_{5/2}(x) = \sqrt{\dfrac{2}{\pi x}}\left\{\dfrac{3-x^2}{x^2}\sin x - \dfrac{3}{x}\cos x\right\}$

(ii) $J_n''(x) = \dfrac{1}{4}\left[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)\right]$

2C. (i) Using the annihilator method solve $y'' + y' = 4x^2 e^x$

(ii) Suppose ϕ is a function having continuous derivative on $0 \le x < \infty$ such that $\phi'(x) + 2\phi(x) \le 1$ for all x and $\phi(0) = 0$. Show that $\phi(x) \le \dfrac{1}{2}$ for $x \ge 0$.

3A.(a) Show that the functions φ_1, φ_2, defined by $\varphi_1 = x^2$, $\varphi_2 = x\,|x|$, are linearly independent for $-\infty < x < \infty$.

(b) Compute the Wronskian of these functions. Compare the two results and explain your answer.

3B. Solve in series $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$.

3C Prove that (1) $H_n'(x) = 2n\,H_{n-1}(x)$

(2) $2x\,H_n(x) = 2n\,H_{n-1}(x) + H_{n+1}(x)$

4A. If $\varphi_1(x)$ is a solution of a differential equation $y'' + a_1(x)\,y' + a_2(x)\,y = 0$ then show that $\varphi_2(x) = \varphi_1(x)\,f(x)$ is a solution of this equation provided $f'(x)$ satisfies the equation $(\varphi_1^2\,y)' + a_1(x)\,(\varphi_1^2\,y) = 0$

4B Find the eigen values and the eigen functions of $y'' + \lambda y = 0$, $y(0) = y(\pi) = 0$.

4C If φ_1, φ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0 then prove that $W(\varphi_1, \varphi_2)(x) = e^{-a_1(x - x_0)}\,W(\varphi_1, \varphi_2)(x_0)$

5A. Solve by method of variation of parameters $y''' + y'' + y' + y = 1$

5B. (i) Show that $\int_{-1}^{1} x^2\,P_{n-1}(x)\,P_{n+1}(x)\,dx = \frac{2n\,(n+1)}{(2n-1)(2n+1)(2n+3)}$

(ii) Prove that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\left(x^n e^{-x}\right)$.

5C. Find the solution of $x^2\,y'' + 9x\,y' + 12\,y = 0$ by series method.

6A. Obtain the series solution of $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)\,y = 0$, n is a real number.

6B. Let φ_1, φ_2 be be two solutions of $L(y) = y'' + a_1\,y' + a_2 y = 0$. Then prove that φ_1, φ_2 are linearly independent on an interval I if and only if $W(\varphi_1, \varphi_2)(x) \neq 0$.

6C. Prove that $\int_0^1 x\,J_n(\alpha x)\,J_n(\beta x)\,dx = \begin{cases} 0, & \alpha \neq \beta \\ \frac{1}{2}\left[J_{n+1}(\alpha)\right]^2, & \alpha = \beta \end{cases}$ where α, β are the roots of $J_n(x) = 0$.

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