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MANIPAL UNIVERSITY, MANIPAL
FIRST SEMESTER M.SC (APPLIED MATHEMATICS & COMPUTING)
END SEMESTER EXAMINATION – NOVEMBER / DECEMEBR 2015

SUBJECT: REAL ANALYSIS (MAT 605)

Time : 3 Hrs. **Max.Marks : 50**

✍ Note : a) Answer any FIVE full questions.
b) All questions carry equal marks (3 + 3 + 4)

1A. Show that a subset E of the real line R is connected if and only if it has the following property: If $x \in E$, $y \in E$, and $x < z < y$, then $z \in E$.

1B. Show that every neighbourhood is an open set.

1C. Show that the set of real numbers R has Archimedean property and hence show that there exists a rational number between any two real numbers.

2A. Show that the sequence of functions $\{f_n\}$ defined on E, converges uniformly on E if and only if for every $\varepsilon > 0$, there exists an integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies $|f_n(x)-f_m(x)| < \varepsilon$.

2B. Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then show that f(X) is compact.

2C. Let $e = \sum_{n=0}^{\infty} \frac{1}{n!}$. Show that $2 < e < 3$. Also show that e is irrational.

3A. Let $\{p_n\}$ be a sequence in a metric space X. If $p \in X$, $p' \in X$, and if $\{p_n\}$ converges to p and to p', then show that $p' = p$.

3B. Obtain the circle of convergence and radius of convergence for the series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} z^{2n}$

3C. Suppose K is compact, and

(i) $\{f_n\}$ is a sequence of continuous functions on K,

(ii) $\{f_n\}$ converges pointwise to a continuous function f on K,

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, n = 1, 2, 3, …..

Then show that $f_n \to f$ uniformly on K.

4A. Define a complete metric space. In any metric space X, show that every convergent sequence is a Cauchy sequence.

4B. If X is a complete metric space, and if ϕ is a contraction of X into X, then show that there exists one and only one $x \in X$ such that $\phi(x) = x$.

4C. Show that a mapping of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

5A. If P^* is a refinement of a partition P then show that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.

5B. Show that $f \in R(\alpha)$ on [a, b] if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(p, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

5C. If $f_1 \in R(\alpha)$ and $f_2(\alpha)$ on [a, b] then show that $f_1 + f_2 \in R(\alpha)$ and that $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.

6A. Define a compact set. If F is a closed subset and K is compact subset of a metric space X, show that $F \cap K$ is compact.

6B. Let $\{E_n\}$, n = 1, 2, 3, … , be a sequence of countable sets, and put $S = \bigcup_{n=1}^{\infty} E_n$. Show that S is countable.

6C. Let X be a metric space. Let $Y \subset X$. Show that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some subset G of X.

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