Reg.No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

THIRD SEMESTER M.Sc(APPLED MATHEMATICS & COMPUTING)
End Semester Makeup Examination January-2016
SUB: GRAPH THEORY
(REVISED CREDIT SYSTEM)

Time: 3 Hrs. **Max. Marks:50**

Note: a). Answer any FIVE full questions. b). All questions carry equal marks

1. (a) Show that for any graph G, either G or $\bar{G}$ is connected. Also show that if for a connected graph G, $diam(G) \geq 3$, then $diam(\bar{G}) \leq 3$ and hence show that diameter of a self complementary graph is either 2 or 3. (4)

   (b) Show that every planar graph is five colourable. (3)

   (c) If G is a connected graph on $p(\geq 3)$ vertices with $\delta \geq {}^{p}/_{2}$, then show that G is Hamiltonian. (3)

2. (a) Define vertex connectivity of a graph. Show that a (p, q) graph G with $p \geq 3$ is 2-connected if and only if any two vertices of G are connected by atleast two vertex disjoint paths. (4)

   (b) Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path. (3)

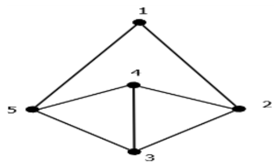
   (c) Show that a (p, q)graph G is a tree if and only if $p = q + 1$ and G is connected. (3)

3. (a) Define Ramsey number $r(m, n)$. Show that $r(k, k) \geq 2^{\frac{k}{2}}$. (4)

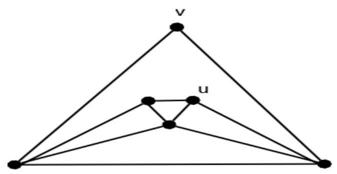
   (b) Show with usual notation that, if G is simple graph then the chromatic polynomial $\pi_k(G)$ satisfies the relation $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$. (3)

   (c) For any (p, q) graph G with line graph $L(G)$, show that $A\big(L(G)\big) = B^T B - 2I_q$ where B is the incidence matrix of G and $A(L(G))$ is the adjacency matrix of $L(G)$. (3)

4. (a) Find the chromatic polynomial of the graph given below. (4)



(b) Write the adjacency matrix of the following graph and hence find number of walks of length three between u and v. (3)



(c) With usual notation show that (i) $2\sqrt{p} \leq \chi + \bar{\chi} \leq p + 1$, (ii) $p \leq \chi\bar{\chi} \leq \left(\frac{p+1}{2}\right)^2$.

5. (a) Show that complement of a strongly regular graph with parameters (n, r, λ, μ) is also strongly regular. (4)

(b). Show that a connected graph is isomorphic to its line graph if and only if it is a cycle. (3)

(c) Show that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering. (3)

6. (a) With usual notation show that $\alpha_0 + \beta_0 = \alpha_1 + \beta_1 = p$. (4)

(b) Find the number of edges in the graph $G_1 \times G_2$ and $G_1[G_2]$, when $G_1 = K_3$ and $G_2 = C_{4.}$ (3)

(c) Show that in a critical graph, no vertex cut is a clique. Also show that every critical graph is a block. (3)

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