Reg.No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

THIRD SEMESTER M.Sc(APPLED MATHEMATICS & COMPUTING)
End Semester Examination November-2015

SUB: GRAPH THEORY
(REVISED CREDIT SYSTEM)

Time: 3 Hrs. **Max. Marks:50**

Note: Answer any FIVE full questions.

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1. (a) With usual notation show that the Ramsey number $R(m,n)$ satisfies the relation $r(m,n) \leq r(m-1,n) + r(m,n-1)$. Hence show that $r(m,n) \leq {}^{m+n-2}C_{m-1}$. (4)

   (b) Let G_1be a (p_1,q_1) and G_2be a (p_2,q_2) graphs. Then, define $G_1 \times G_2$ and $G_1 + G_2$. Find number of vertices and edges in both. (3)

   (c) Show that if a regular connected graph G is of diameter 3 then $\bar{G}$ is of diameter 2. Hence show that there does not exist a regular self-complementary graph of diameter 3. (3)

2. (a) With usual notation, prove that $\alpha_0 + \beta_0 = \alpha_1 + \beta_1 = p$. (4)

   (b) Show that a graph G is 2-connected if and only if between every two vertices of G there are vertex disjoint paths. (3)

   (c) Show that a (p,q)graph G is a tree if and only if $p = q + 1$ and G is connected. (3)

3. (a) Let G be any graph and let u and v be vertices in G such that $deg(u) + deg(v) \geq r$. Then show that G is Hamiltonian if and only if $G + (u,v)$ is Hamiltonian. (4)

   (b) Let G be a bipartite graph with bipartition (X,Y). Then show that G contains a matching that saturates every vertex of X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. (3)

   (c) Show that the Ramsey number $r(m,n)$ satisfies the inequality $r(n,n) \geq 2^{\frac{n}{2}}$. (3)

3. (a) Let G be a graph with $V(G) = \{v_1, v_2, \dots, v_n\}$. Define an elementary subgraph of G and show that, for adjacency matrix A of G,

$\det A = \sum (-1)^{n-c_1(H)-c(H)} 2^{c(H)}$, where $c(H) and\, c_1(H)$ are the number of components in H which are cycles and edges, respectively and the summation is taken over every elementary subgraph H of G. (4)

(b) Let $\{S_1, S_2, \dots, S_n\}$be any partition of the set of integers $\{1,2,\dots,r_n\}$ where $r_n = r(k_1, k_2, .., k_n)$where $k_i = 3$ for all i. Then show that, for some i, S_i contains three integers x, y and z satisfies the relation $x + y = z$. (3)

(c) Show that if G is a tree on n vertices, then $\pi_k(G) = k(k-1)^{n-1}$.

Also show that, if G is a cycle with length n then,

$\pi_k(G) = (k-1)^n + (-1)^n(k-1)$. (3)

5. (a) State and prove five colour theorem. (4)

(b) Let G be a graph on n vertices. Columns $j_1, j_2, \dots, j_k$ of the $(0,1,-1)$ incidence matrix $Q(G)$ are linearly independent if and only if the corresponding edges of G induce an acyclic graph. (3)

(c) Let G be a tree on n vertices and $Q(G)$ be the $(0,1,-1)$ incidence matrix of G. Then show that any submatrix of $Q(G)$ of order $(n-1)$ is nonsingular. (3)

6. (a) With usual notation show that

(i) $2\sqrt{p} \leq \chi + \bar{\chi} \leq p + 1$

(ii) $p \leq \chi\bar{\chi} \leq \left(\frac{p+1}{2}\right)^2$. (4)

(b) show that a connected graph is isomorphic to its line graph if and only if it is a cycle. (3)

(c) For any (p, q) graph G with line graph $L(G)$, show that $A(L(G)) = B^T B - 2I_q$ where B is the incidence matrix of G and $A(L(G))$ is the adjacency matrix of $L(G)$. (3)

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