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# MANIPAL UNIVERSITY, MANIPAL

#### THIRD SEMESTER M.SC (APPLIED MATHEMATICS & COMPUTING) END SEMESTER EXAMINATION – NOVEMBER, 2015

### SUB : OPTIMIZATION METHODS-I (MAT 701)

#### (REVISED CREDIT SYSTEM)

| Time: 3 Hrs. |  |
|--------------|--|
|--------------|--|

Max. Marks : 50

## Note : a) Answer any FIVE full questions. b) All questions carry equal marks.

1A. A firm produces three products A, B, & C. It uses two types of raw materials I & II of which 5000 and 7500 units are available. The raw material requirements per unit of the products are given below

| Raw       | Requirement / Unit of products |   |   |  |  |  |
|-----------|--------------------------------|---|---|--|--|--|
| materials | А                              | В | С |  |  |  |
| Ι         | 3                              | 4 | 5 |  |  |  |
| II        | 5                              | 3 | 5 |  |  |  |

The labour time for each unit of product A is twice that a product B and three times as that of product C. The entire labour force of the firm can produce the equivalent of 3000 units. The minimum demand for the three products are 600, 650 and 500 units. Also the ratio of the number of units produced must be 2:3:4. Assuming the profits per unit of A, B and C are Rs. 50, 50 and 80 respectively, formulate the problem mathematically to determine the number of units of each of the product which will maximize the total profit.

1B. A company possesses two manufacturing plants which can produce 3 products X, Y and Z from a common raw material. The data on production per hour and costs together with current orders in hand for each product, is as follows.

| Plants         | Products |    |    | Operating cost per |
|----------------|----------|----|----|--------------------|
|                | Χ        | Y  | Ζ  | hour (In Rs.)      |
| А              | 2        | 4  | 3  | 9                  |
| В              | 4        | 3  | 2  | 10                 |
| Orders on hand | 50       | 24 | 60 |                    |

Use graphical method to determine the number production hours needed to fulfill the orders at minimum total cost.

1C. Derive the condition for the feasibility of a solution of a LPP in the standard form using simplex method. (3+4+3)

2A. A particular product is manufactured in factories A, B, C & D and are sold at centres 1,2,3. The relevant data are given below :

| Factory | Cost<br>/unit(Rs) | Capacities | Sales  | Sale       | Demand |
|---------|-------------------|------------|--------|------------|--------|
|         | /unit(Rs)         |            | Centre | Price/Unit |        |
| А       | 12                | 100        | 1      | 15         | 120    |
| В       | 15                | 20         | 2      | 14         | 140    |
| С       | 11                | 60         | 3      | 16         | 60     |
| D       | 13                | 80         |        |            |        |

Find the optimal sales distribution. Use VAM to obtain the initial basic solution.

2B. Solve the following LPP by branch and bound method Maximize  $Z = 4x_1 + 3x_2$ 

Subject to :

3A. Use two phase method to

3B. A firm produces four products A, B, C & D. There are four operators who are capable of producing any of these four products. The firm records 8 hours a day and allows 30 minutes for lunch. The processing time in minutes and profit for each the products are given below.

Find the best assignment of products to operators.

(6 + 4)

4A. Reduce the game defined by the following pay off matrix to a 2 X 2 game and then solve

Player II  

$$B_{1} \quad B_{2} \quad B_{3} \quad B_{4}$$
Player I  

$$A_{1} \quad \begin{bmatrix} 3 & 1 & 3 & 2 \\ 2 & 7 & -5 & 1 \\ A_{3} & \\ A_{4} & \begin{bmatrix} 3 & 4 & -1 & 2 \\ 3 & 3 & -2 & 2 \end{bmatrix}$$

4B.

A project has following activities precedence relations and time estimates

| Activity | Immediate   | Duration ( in weeks) |              |             |  |  |  |
|----------|-------------|----------------------|--------------|-------------|--|--|--|
|          | predecessor | Optimistic           | Most likely` | Pessimistic |  |  |  |
| А        | -           | 4                    | 5            | 12          |  |  |  |
| В        | -           | 2                    | 9            | 10          |  |  |  |
| С        | -           | 4                    | 5            | 12          |  |  |  |
| D        | В           | 8                    | 10           | 12          |  |  |  |
| E        | A,D         | 3                    | 4            | 11          |  |  |  |
| F        | В           | 3                    | 4            | 5           |  |  |  |
| G        | В           | 4                    | 5            | 12          |  |  |  |
| Н        | С           | 3                    | 4.5          | 9           |  |  |  |
| Ι        | С           | 1                    | 3            | 11          |  |  |  |
| J        | С           | 6                    | 8            | 10          |  |  |  |
| K        | E,F         | 1.5                  | 2.5          | 6.5         |  |  |  |
| L        | G,H,K       | 7                    | 9            | 11          |  |  |  |
| М        | E,F         | 2                    | 5.5          | 6           |  |  |  |
| Ν        | E,F         | 4                    | 5            | 12          |  |  |  |
| 0        | I,L,M       | 1                    | 3            | 11          |  |  |  |

Draw a network and find the critical path. What should be the due date to have 0.90 probability of completion of the project. Given,  $\phi^{-1}(0.90) = 1.28$ .

(4 + 6)

5A. Solve the game defined by the following pay off matrix by simplex method

Player *II*  

$$B_1 \quad B_2 \quad B_3$$
  
 $A_1 \begin{bmatrix} 3 & 4 & -2 \\ -3 & 0 & 1 \\ A_3 \end{bmatrix}$   
Player I  $A_2 \begin{bmatrix} -3 & 0 & 1 \\ -1 & -4 & 2 \end{bmatrix}$ 

**5B.** A project has following activities, precedence relations and time estimates.

| Activities            | Α | В | С | D   | E   | F   | G   | Н     |
|-----------------------|---|---|---|-----|-----|-----|-----|-------|
| Immediate predecessor |   | - | I | A,B | B,C | A,B | C   | D,E,F |
| Duration ( in days)   | 4 | 8 | 6 | 11  | 6   | 4   | 6   | 3     |
| Activities            | Ι | J | K | L   | М   |     | N   |       |
| Immediate predecessor | D | G | G | H,J | K   |     | I,L |       |
| Duration ( in days)   | 4 | 8 | 5 | 10  | 5   |     | 7   |       |

Draw a network, find the critical path. Compute total and free floats for all noncritical activities.

6A. Given a LPP, Minimize z = x-3y+2zSubject to  $3x-y+2z \le 7$   $-2x+4y \le 12$   $-4x+3y+8z \le 10$   $x,y,z \ge 0$ (5+5)

Solve the LPP by Simplex method and discuss the effect of change in availability of resources without change in the optimal value.

| Activity | Preceding  | Time (in weeks) |       | Cost   | : (Rs.) |  |
|----------|------------|-----------------|-------|--------|---------|--|
|          | Activities | Normal          | Crash | Normal | Crash   |  |
| А        | -          | 3               | 2     | 18000  | 19000   |  |
| В        | -          | 8               | 6     | 600    | 1000    |  |
| С        | В          | 6               | 4     | 10000  | 12000   |  |
| D        | В          | 5               | 2     | 4000   | 10000   |  |
| Е        | А          | 13              | 10    | 3000   | 9000    |  |
| F        | А          | 4               | 4     | 15000  | 15000   |  |
| G        | F          | 2               | 1     | 1200   | 1400    |  |
| Н        | C,E,G      | 6               | 4     | 3500   | 4500    |  |
| Ι        | F          | 2               | 1     | 7000   | 8000    |  |

6B. A project has following activities, time and cost estimates.

Draw a network and find the critical path. If a dead line of 19weeks is imposed for completion, what activities will be crashed ? What is the total cost after crashing?

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(6+4)