

Max. Marks: 50

NOTE: (a) Answer any FIVE full questions. (b) All questions carry equal marks.

1. (a) If $\psi(r, \theta, \phi) = 2rsin\theta + r^2 cos\phi$, find $\nabla \psi$ and $\nabla^2 \psi$ in spherical coordinate system (b) Obtain Laurentz expansion for

$$f(z) = \frac{1}{4+z^2}$$

which is valid in the region |z - 2i| > 4

- 2. (a) Find a complex Fourier series for f(x) = x in the range -2 < x < 2.
 - (b) State and prove Cauchy residue theorem
- Show that the Schrodinger equation of a quantum harmonic oscillator takes the form of 3. (a) Hermite differential equation
 - The displacement of a damped harmonic oscillator as a function of time is given by, (b) $f(t) = e^{-t/\tau} sin(\omega_0 t)$ for $t \ge 0$ Obtain Fourier transform of f(t)
- 4. (a) Diagonalize the matrix given below

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 3 \end{bmatrix}$$

(b) Evaluate the following integral using <u>residue theorem</u>

$$\int_{0}^{2\pi} \frac{d\theta}{1 + \cos\theta/2}$$

- 5. (a) Obtain an isomorphic representation of $C3_v$ group.
 - Find the recurrence relation for Laguerre differential equation and obtain first four (b) Laguerre polynomials
- (a) A coordinate system (u, v, w) is related to Cartesian coordinate system by the relations 6. x = vw, y = uw, z = uv
 - Find the components of metric tensor and its reciprocal in this coordinate system.
 - Find Fourier transform of the exponential decay function $f(t) = Ae^{-\beta t}$ (where $\beta > 0$) (b) for $t \ge 0$