Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
Manipal University

FIRST SEMESTER M.TECH (DEAC) DEGREE END SEMESTER EXAMINATION
NOV/DEC 2015
SUBJECT: DETECTION AND ESTIMATION THEORY (ECE - 507)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates
- Answer **ANY FIVE** full questions.
- Missing data may be suitably assumed.

1A. Consider a binary hypothesis testing problem with the following conditional density functions:

$$p_{R/H_0}(r/H_0) = e^{-r},\ r \ge 0, \quad p_{R/H_1}(r/H_1) = 2e^{-2r}, r \ge 0$$

If the two hypothesis are equally likely,

i) Formulate an optimum decision rule assuming the cost for correct decision is zero and for a wrong decision is one.
ii) Obtain the probability of error

1B. Consider the following conditional density functions in the context of a binary hypothesis testing problem

$$p_{R/H_1}(r/H_1) = \frac{1}{\sqrt{4\pi}} \exp\left(\frac{-(r-4)^2}{4}\right) \text{ and } p_{R/H_0}(r/H_0) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-r^2}{2}\right)$$

If the cost for a wrong decision is 1 and the cost for a correct decision is 0, determine the minimax criterion based detection threshold.

1C. Explain briefly the Neyman-Pearson criteria for hypothesis testing

(5+3+2)

2A. Consider K observations such that $R_k=m+N_k$, k=1, 2, 3, ...K, where m is unknown and N_k's are statistically independent zero mean Gaussian random variables with unknown variance σ^2.

i) Find the estimates for m and σ^2.
ii) Is the estimator of m an efficient estimator?

2B. Derive the Cramer-Rao bound for estimators of non-random parameters.

2C. Compare a MAP estimator with a M-L estimator

(5+3+2)

3A. Using the Gram-Schmidt orthogonalization procedure, obtain a generalized Fourier series expansion of the signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ shown in figure Q3A. Also, sketch the signal constellation.

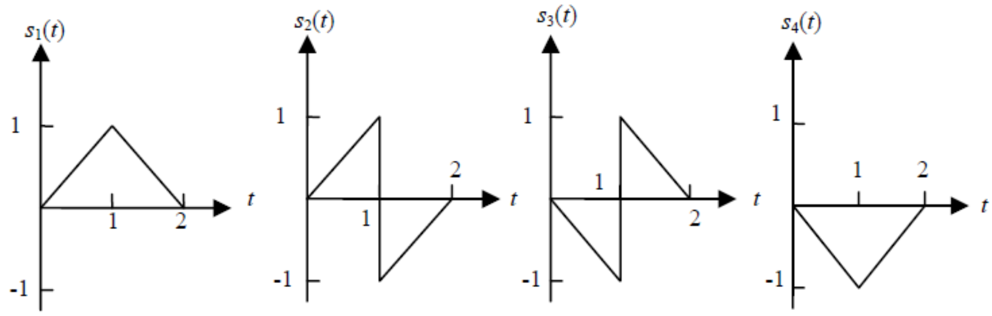

Figure Q3A

3B. Describe in detail the series expansion of a random process.

3C. Show that a wide sense stationary Gaussian random process can be expressed in a KLSE format using any arbitrary set of basis functions

(5+3+2)

4A. Consider a general binary detection problem as given below.

$$H_1: R(t) = s_1(t)+W(t) \quad 0 < t < T$$

$$H_0: R(t) = s_0(t)+W(t) \quad 0< t < T$$

Here, $s_1(t)$ and $s_0(t)$ are known waveforms and $W(t)$ is AWGN of two sided power spectral density $N_o/2$. Derive and optimum receiver for this problem and draw its block diagram

4B. Using the results of 4A above, arrive at expressions for the probability of error for coherent detection of Binary PSK, Binary ASK, and Binary FSK in AWGN

4C. Draw the schematic of a MAP estimator of the phase of a sinusoidal signal

(5+3+2)

5A. Let R_k be the observed random variable such that $R_k=a+bx_k+N_k$, $k=1,2,...K$

The constants x_k, $k=1,2,...K$, are known while the constants a and b are not known. If the random variables N_k are statistically independent and Gaussian distributed with mean zero variance σ^2, obtain the ML estimate of (a, b).

5B. Let $s_1(t) = 1$ and $s_2(t) = t$ be defined on the interval [-1, 1]. Are $s_1(t)$ and $s_2(t)$ orthogonal in the given interval? Determine the constants α and β such that $s_3(t)=1+\alpha t+ \beta t^2$ is orthogonal to both $s_1(t)$ and $s_2(t)$ in the interval [-1, 1].

5C. Show that: i) $E(\Lambda(\bar{r})/H_0)=1$ ii) $E\left(\frac{1-\Lambda(\bar{r})}{\Lambda(\bar{r})}\Big/H_1\right)=0$

(5+3+2)

6A. What is the orthogonality principle in the context of minimum mean square error optimum filtering of a noisy signal? Derive this principle and with the help of this provide the frequency response of an optimum (non-causal) filter.

6B. Let $R(t) = A(t) + N(t)$ in the interval $0 < t < T$ with $N(t)$ being an additive noise process. Obtain an optimum filter for $D(t) = dA(t)/dt$ if $K_{AA}(t) = e^{-2t}$ and $K_{NN}(t) = (N_0/2)\delta(t)$.

6C. In part 6B above, if you were to estimate the second derivative of $A(t)$, what would be the filter frequency response. Generalize this result for n^{th} order differentiation of $A(t)$

(5+3+2)