Reg. No. | | | | | | | | | | |

MANIPAL INSTITUTE OF TECHNOLOGY, MANIPAL 576104
(Constituent College of Manipal University)

FIRST SEMESTER M.TECH(Software Engineering) DEGREE MAKE UP EXAMINATION-JAN 2016

SUBJECT:MATHEMATICAL LOGIC (ICT 523)
(REVISED CREDIT SYSTEM)

TIME:3 HOURS 05/01/2016 MAX.MARKS:50

Instructions to Candidates

- Answer any **FIVE FULL** questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.

1A. Convert the given paragraph in a symbolic sequent and prove it.
If interest rates fall, then the stock market will rise. If interest rates do not fall, then housing starts and consumer spending will fall. Now, consumer spending is not falling. So, its true that housing starts are not falling or consumer spending is not falling, that is, it is false that housing starts and consumer spending are both falling. This means that interest rates are falling, so the stock market will rise.

1B. Prove the validity of the sequent

$$p \to q \vdash ((p \wedge q) \to p) \wedge (p \to (p \wedge q)).$$

1C. Check whether the following Horn formulas are satisfiable:

i) $(\top \to q) \wedge (\top \to s) \wedge (w \to \bot) \wedge (p \wedge q \wedge s \to \bot) \wedge (v \to s) \wedge (\top \to r) \wedge (r \to p)$,

ii) $(\top \to q) \wedge (\top \to s) \wedge (w \to \bot) \wedge (p \wedge q \wedge s \to v) \wedge (v \to s) \wedge (\top \to r) \wedge (r \to p)$.

[5+3+2]

2A. Find appropriate predicates and their specification to translate the following into predicate logic:

i) All red things are in the box.

ii) Only red things are in the box.

iii) No animal is both a cat and a dog.

iv) Every prize was won by a boy.

v) A boy won every prize.

2B. Let $\mathcal{F}$ be $\{d, f, g\}$, where d is constant, f a function symbol with arity two and g a function symbol with arity three.

i) Which of the following strings are terms over $\mathcal{F}$?

a) $g(d, d)$

b) $f(x, g(y, z), d)$

c) $g(x, f(y, z), d)$

d) $g(x, h(y, z), d)$

e) $f(f(g(d, x), f(g(d, x), y, g(y, d)), g(d, d)), g(f(d, d, x), d), z)$.

ii) The length of a term over $\mathcal{F}$ is the length of its string representation, where we count all commas and parentheses. List all variable-free terms over $\mathcal{F}$ of length less than 10.

iii) The height of a term over $\mathcal{F}$ is defined as 1 plus the length of the longest path in its parse tree. List all variable-free terms over $\mathcal{F}$ of height less than 4.

2C. Draw the parse tree for the following CTL formulas:

i) $AG(q \to EGr)$

ii) $A[p\ U\ EFr]$

iii) $EFEGp \to AFr$

iv) $AG(p \to A[p\ U\ (\neg p \land A[\neg p\ U\ q])])$.

[5+3+2]

3A. Let Γ be the set of formulas

$$\begin{aligned}\Gamma = \Big\{ & C(p_1 \lor p_2 \lor p_3),\\ & C(p_1 \to K_2 p_1), C(\neg p_1 \to K_2 \neg p_1),\\ & C(p_1 \to K_3 p_1), C(\neg p_1 \to K_3 \neg p_1),\\ & C(p_2 \to K_1 p_2), C(\neg p_2 \to K_1 \neg p_2),\\ & C(p_2 \to K_3 p_2), C(\neg p_2 \to K_3 \neg p_2)\\ & C(p_3 \to K_1 p_3), C(\neg p_3 \to K_1 \neg p_3)\\ & C(p_3 \to K_2 p_3), C(\neg p_3 \to K_2 \neg p_3) \Big\}.\end{aligned}$$

Prove the sequent $\Gamma, C(p_2 \lor p_3), C(\neg K_2 p_2 \land \neg K_2 \neg p_2) \vdash K_3 p_3$.

3B. Consider a formula scheme $\Box\phi \to \Box\Box\phi$, as per the correspondence theory prove that its accessibility relation R has transitive property.

3C. Consider the Kripke model $\mathcal{M}$ depicted in Figure Q.3C. For each of the following determine whether it holds:

i) $a \Vdash q$

ii) $a \Vdash \Box\Box q$

iii) $a \Vdash \Box\Diamond\neg q$

iv) $c \Vdash \Box\bot$.

[5+3+2]

4A. Consider the wise-men puzzle. Justify your answers.

i) Each man is asked the question 'Do you know the color of your hat?' Suppose that the first man says 'no,' but the second one says 'yes.' Given this information together with the common knowledge, can we infer the color of his hat?

ii) Can we predict whether the third man will now answer 'yes' or 'no'?

iii) What would be the situation if the third man were blind?

4B. Prove sequent $\vdash_K \Box(p \to q) \vdash \Diamond p \to \Diamond q$.

4C. Using the natural deduction rules for $KT45^n$, prove the validity of $K_i Cp \leftrightarrow Cp$.

[5+3+2]

5A. Give Backus Naur form for branching tree logic and define the relation $\mathcal{M}, s \vDash \phi$ by structural induction on ϕ.

5B. Consider the system of Figure Q.5B. For each of the formula ϕ:

i) Ga

ii) $a\ U\ b$

iii) $a\ U\ X(a \wedge \neg b)$

iv) $X\neg b \wedge G(\neg a \vee \neg b)$

v) $X(a \wedge b) \wedge F(\neg a \wedge \neg b)$.

Find a path from the initial state q_3 which satisfies ϕ.

5C. Prove the sequent $\forall x \forall y P(x, y) \vdash \forall u \forall v P(u, v)$.

[5+3+2]

6A. Does $\vDash \phi$ hold for the ϕ below? Justify your answer.

i) $(p \to q) \vee (q \to r)$

ii) $((q \to (p \vee (q \to p))) \vee \neg(p \to q)) \to p$.

6B. Describe basic labelling algorithms with reference to CTL model checking.

6C. You are asked to prove the validity of a sequent $\forall x(P(x) \to Q(x)), \exists x P(x) \vdash \exists x Q(x)$. Suppose your proof is as follows:

1.	$\forall x(P(x) \to Q(x))$	Premise
2.	$\exists x P(x)$	Premise
3.	$x_0, P(x_0)$	Assumption
4.	$P(x_0) \to Q(x_0)$	$\forall x e$, 1
5.	$Q(x_0)$	$\to e$ 3,4
6.	$Q(x_0)$	$\exists x e$ 2,3-5
7.	$\exists x Q(x)$	$\exists x i$ 6

Is this proof correct? If not then what is wrong with this proof?

[5+3+2]

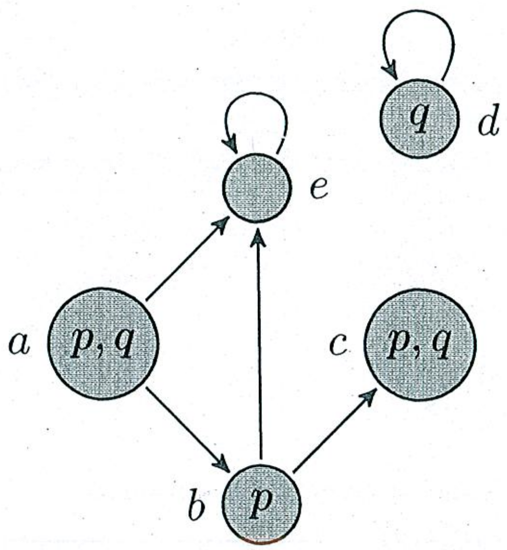

Figure: Q.3C

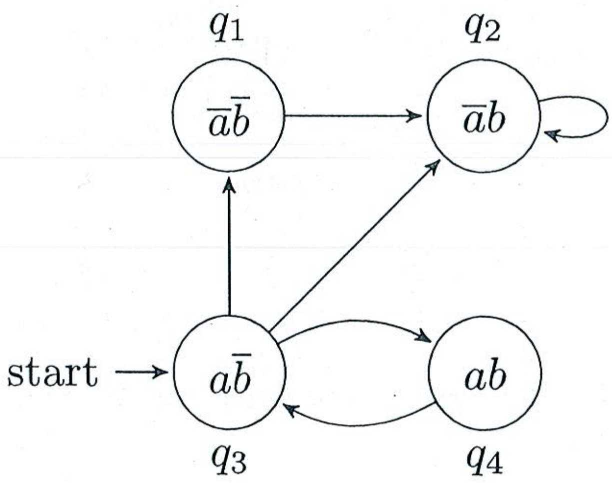

Figure: Q.5B