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MANIPAL INSTITUTE OF TECHNOLOGY, MANIPAL 576104
(Constituent College of Manipal University)

FIRST SEMESTER M.TECH(Software Engineering) DEGREE END SEMESTER EXAMINATION-NOV/DEC 2015

SUBJECT:MATHEMATICAL LOGIC (ICT 523)
(REVISED CREDIT SYSTEM)

TIME:3 HOURS 03/12/2015 MAX.MARKS:50

**Instructions to Candidates**

- Answer any **FIVE-FULL** questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.

1A. Brown, Jones, and Smith are suspected of a crime. They testify as follows:

- Brown: Jones is guilty and Smith is innocent.
- Jones: If Brown is guilty then so is Smith.
- Smith: I am innocent, but at least one of the others is guilty.

Let $b$, $j$, and $s$ be the statements "Brown is innocent, "Jones is innocent, "Smith is innocent.

i) Express the testimony of each suspect as a propositional formula.

ii) Are the three testimonies consistent?

iii) The testimony of one of the suspects follows from that of another. Which from which?

iv) Assuming everybody is innocent, who committed perjury?

v) Assuming all testimony is true, who is innocent and who is guilty?

vi) Assuming that the innocent told the truth and the guilty told lies, who is innocent and who is guilty?

1B. Check whether the sequent, $p \to q, s \to t \vdash p \vee s \to q \wedge t$ is valid, if yes then prove it.

1C. Check whether the following Horn formulas are satisfiable:

i) $(p \wedge q \wedge w \to \bot) \wedge (t \to \bot) \wedge (r \to p) \wedge (\top \to r) \wedge (\top \to q) \wedge (u \to s) \wedge (\top \to u)$,

ii) $(p \wedge q \wedge w \to \bot) \wedge (t \to \bot) \wedge (r \to p) \wedge (\top \to r) \wedge (\top \to q) \wedge (r \wedge u \to w) \wedge (u \to s) \wedge (\top \to u)$.

[5+3+2]

2A. Assume the following predicates:

- $H(x)$ : $x$ is a human
- $C(x)$ : $x$ is a car
- $T(x)$ : $x$ is a truck

- $D(x, y)$: $x$ drives $y$

Write formulas representing the following statements:

i) Everybody drives a car or a truck.
ii) Some people drive both.
iii) Some people dont drive either.
iv) Nobody drives both.
v) Only cars and trucks are driven.

2B. Let $P$ be a predicate with symbol with arity three. Draw the parse tree of

$$\psi \triangleq \neg(\forall x((\exists P(x, y, z)) \wedge (\forall z P(x, y, z))))$$

i) Indicate the free and bound variables in that parse tree.
ii) List all variables which occur free and bound therein.
iii) Compute $\psi[t/x]$, $\psi[t/y]$ and $\psi[t/z]$, where $t \triangleq g(f(g(y, y)), y)$. Is $t$ free for $x$ in $\psi$; free for $y$ in $\psi$; free for $z$ in $\psi$?

2C. Prove the sequent, $\exists x \forall y P(x, y) \vdash \forall y \exists x P(x, y)$.

[5+3+2]

3A. Write Backus Naur Form for LTL and define its semantic satisfaction relation $\models$ using structural induction.

3B. Consider a system of mutual exclusion, wherein concurrent processes share a resource, it may be necessary to ensure that they do not have access to it at the same time. The system identify certain critical section of each process and arrange that only one process can be in critical section at a time. The critical section should include all the access to the shared resource. Give a model for the mutual exclusion. State and write following properties for your model in LTL

i) Safety
ii) Liveness
iii) Non-blocking, and
iv) No strict sequencing.

3C. Draw parse tree for the LTL formulas:

i) $Fp \wedge Gq \rightarrow pWr$
ii) $F(p \rightarrow Gr) \vee \neg qUp$
iii) $pW(qWr)$
iv) $GFp \rightarrow F(q \vee s)$.

[5+3+2]

4A. Let $\Gamma$ be the set of formulas

$$\Gamma = \Big\{C(p_1 \vee p_2 \vee p_3),$$
$$C(p_1 \to K_2 p_1), C(\neg p_1 \to K_2 \neg p_1),$$
$$C(p_1 \to K_3 p_1), C(\neg p_1 \to K_3 \neg p_1),$$
$$C(p_2 \to K_1 p_2), C(\neg p_2 \to K_1 \neg p_2),$$
$$C(p_2 \to K_3 p_2), C(\neg p_2 \to K_3 \neg p_2)$$
$$C(p_3 \to K_1 p_3), C(\neg p_3 \to K_1 \neg p_3)$$
$$C(p_3 \to K_2 p_3), C(\neg p_3 \to K_2 \neg p_3)\Big\}.$$

Prove the sequent, $\Gamma, C(\neg K_1 p_1 \wedge \neg K_1 \neg p_1) \vdash C(p_2 \vee p_3)$.

4B. Consider a formula scheme, $\Box\phi \to \phi$, from correspondence theory prove that the accessibility relation $R$ for this formula scheme has the reflexive property.

4C. Consider the Kripke model depicted in Figure Q.4C. Find for each of the following a world which satisfies it:

i) $\Diamond p \vee \Diamond q$

ii) $\Diamond(p \vee \Diamond q)$.

[5+3+2]

5A. Determine which of the following hold in the Kripke model of Figure Q.5A and justify your answer.

i) $x_3 \Vdash K_1(p \vee q)$

ii) $x_1 \Vdash K_2 q$

iii) $x_3 \Vdash E(p \vee q)$

iv) $x_1 \Vdash Cq$

v) $x_1 \Vdash D_{\{1,3\}} p$

5B. Prove the following sequent over the basic modal logic $K$

$$\vdash_K \Box(p \to q) \wedge \Box(q \to r) \to \Box(p \to r).$$

5C. Explain why $C_G\phi \to C_G C_G \phi$ and $\neg C_G \phi \to C_G \neg C_G \phi$ are valid.

[5+3+2]

6A. Find the proofs for

i) $P(b) \vdash \forall x(x = b \to P(x))$

ii) $\exists x \exists y(H(x,y) \vee H(y,x)), \neg\exists x H(x,x) \vdash \exists x \exists y \neg(x = y)$.

6B. Write a function SAT, which takes a CTL formula as input and returns the set of states satisfying the formula.

6C. Run labelling algorithm on second model of mutual exclusion applied to the formula $E[\neg c_1 \; U \; c_2]$.

[5+3+2]

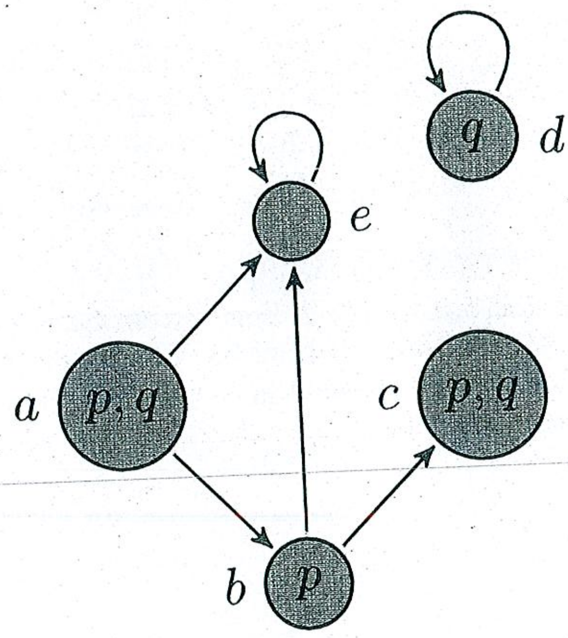


Figure: Q.4C

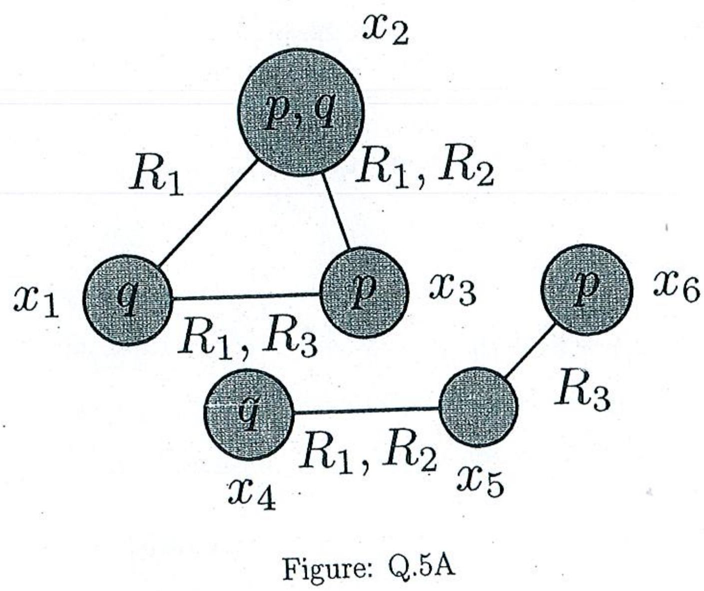


Figure: Q.5A