



Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)

FIRST SEMESTER M.TECH (CONTROL SYSTEMS) END SEMESTER

END SEMESTER EXAMINATIONS, NOV/DEC 2015

SUBJECT: DYNAMICS OF LINEAR SYSTEMS (ICE503)

Time: 3 Hours

MAX. MARKS: 50

Instructions to Candidates:

- ❖ Answer **ANY FIVE FULL** questions.
- ❖ Missing data may be suitably assumed.

1A. Obtain a state model for the following transfer function in diagonal canonical form **4**

$$\frac{y(s)}{u(s)} = \frac{s^3 + 10s^2 + 17s + 8}{(s+6)(s+2)(s+3)}.$$

1B. Derive the transfer function for the following state model. **3**

$$\dot{X}(t) = \begin{bmatrix} -3 & -2 \\ -2 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U(t); \quad Y(t) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} X(t)$$

1C. Define eigen values, eigen vectors, generalized eigen vectors. **3**

2A. Transform the following continuous time state model into diagonal form. **4**

$$\dot{X}(t) = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -2 & 0 \\ 1 & 0 & -1 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} U(t); \quad y(t) = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \bar{X}(t).$$

2B. Derive the state variable representation of armature controlled servo motor using physical variables. **3**

2C. Using Cayley Hamilton method determine the state transition matrix for $A = \begin{bmatrix} 0 & 3 \\ 4 & 4 \end{bmatrix}$. **3**

3A. Using Laplace transform, obtain the time response of the following system **5**

$$\dot{X}(t) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U(t); \text{ where } U(t) \text{ is a unit step input occurring at } t=0 \text{ and } X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

3B. Derive the pulse transfer function **3**

$$X(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix} X(k) + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} U(K); \quad Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \bar{X}$$

3C. State Lyapunov stability and instability theorems. **2**

- 4A.** Consider the system $\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -11 \\ 6 & 1 & -6 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U(t); Y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \dot{X}(t)$ **6**
- Compute the state feedback gain matrix so that the control law $u = -kX(t)$ places the closed loop poles at $-1 \pm j2, -5$.
- 4B.** Given the state equations of a digital control system **4**
- $$x_1(k+1) = x_2(k)$$
- $$x_2(k+1) = x_3(k)$$
- $$x_3(k+1) = 0.4x_2(k) + 0.3x_3(k) + u(k)$$
- $$y(k) = x_1(k)$$
- Transform the state equations into diagonal form and investigate the controllability and observability of the system.
- 5A.** Discretize the continuous time system with sampling period 0.1 second. **5**
- $$\dot{X}(t) = \begin{bmatrix} -3 & 2 \\ 4 & -5 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U(t).$$
- 5B.** For a certain system, when $x(0) = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, then $x(t) = \begin{bmatrix} e^{-3t} \\ -3e^{-3t} \end{bmatrix}$ and when $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then $x(t) = \begin{bmatrix} e^t \\ e^t \end{bmatrix}$. Determine the system matrix A. **3**
- 5C.** Define the concept of complete state controllability and observability for continuous time systems. How are they determined? **2**
- 6A.** The system described by the pulse transfer function **6**
- $$G(z) = \frac{z^3 + 8z^2 + 17z + 8}{(z+1)(z+2)(z+3)}$$
- Obtain the state model (i) controllable canonical form (ii) Observable canonical form.
- 6B.** A unity feedback system is characterized by the open-loop transfer function **4**
- $$G(z) = \frac{0.2385(z + 0.876)}{(z-1)(z-0.2644)}$$
- ; for $T = 0.2$ sec; Determine the steady state errors for unit step, unit ramp and unit acceleration input.
