Reg.No.										

MANIPAL INSTITUTE OF TECHNOLOGY
(A constituent institute of Manipal University)

I SEMESTER M.TECH DEGREE (ADVANCED THERMAL POWER & ENERGY SYSTEMS)
END SEMESTER EXAMINATIONS, NOV/DEC 2015

SUBJECT: FINITE ELEMENT METHODS FOR HEAT & FLUID FLOW (MME – 547)
(REVISED CREDIT SYSTEM)

Time: 3 Hours. MAX.MARKS: 50

Instructions to Candidates:
- Answer **ANY FIVE FULL** questions.
- Missing data, if any, may be suitably assumed.

Q.1A What is meant by Simple Natural and Area Coordinates? Explain with neat sketches. (02)

Q.1B Using Variational Calculus formulation, prove that for a general three dimensional conduction steady state heat transfer having different kinds of thermal loadings, the finite element equation is given by, (08)

$$[K]\{T\}=\{Q\}$$

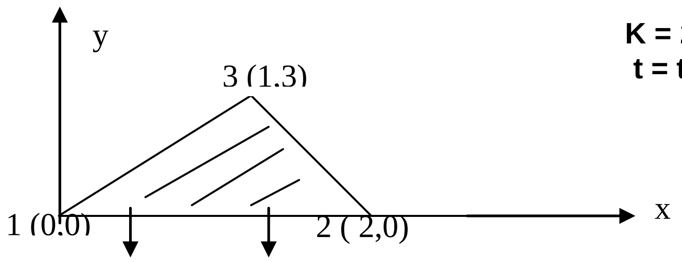
where,

$$[K]=\iiint_V [B]^T[D][B]\,dV+\iint_{S_2} h[N]^T[N]\,ds$$

$$\{Q\}=\iiint_V \dot{q}_g[N]^T dV+\iint_{S_2} q[N]^T ds+\iint_{S_3} hT_\infty[N]^T ds$$

Q.2A Define an Isoparametric Element. Bring out the salient differences between Sub-parametric and Super- Parametric elements. Compare the relative importance each. (04)

Q.2B Determine the nodal Conductance Matrix and Thermal Load vector for a three noded Linear triangular thermal element as given , **Fig 1** below: (06)



K = 20 W/mm.K
t = thickness = 10 mm

Fig. 1

h= 200 W/mm^2.K
T_∞=25^0 C

Q.3A Use the Galerkin formulation to obtain the Thermal Conductance and Load matrices for a slender fin with the open end insulated. Assume a two noded linear steady state heat transfer element. The Governing Differential Equation (with usual notations), is given by, (07)

$$K\frac{d^2T}{dx^2}-\left(\frac{P}{A}\right)h(T-T_\infty)=0$$

Q.3B Determine the Shape functions for the four noded Isoparametric rectangular element. Compute the temperature and the heat fluxes at a location (**ξ**=0.2, η=0.1) in the element as shown in Fig.2. Dimensions are in cm. (03)

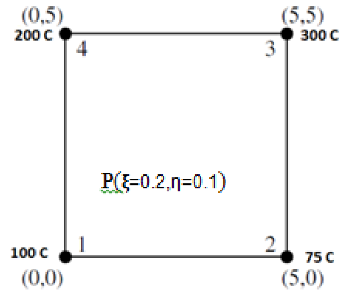


Fig 2

Q.4A A one dimensional quadratic element is used to approximate the temperature distribution in a long fin. The solution gives the temperature as 100^0C, 90^0C and 80^0C at distances of 100 mm, 150mm, 200 mm respectively from the origin. Compute the temperature and its gradient at a location of 120 mm from the origin. (03)

Q. 4B For the composite plane wall having one dimensional steady state heat transfer as shown in **Fig.3**, obtain the finite element equilibrium equations in the form $[K]_{th}\{T\}=\{Q\}_{loads}$. (07)

Use discrete system analysis using energy balance at each node.

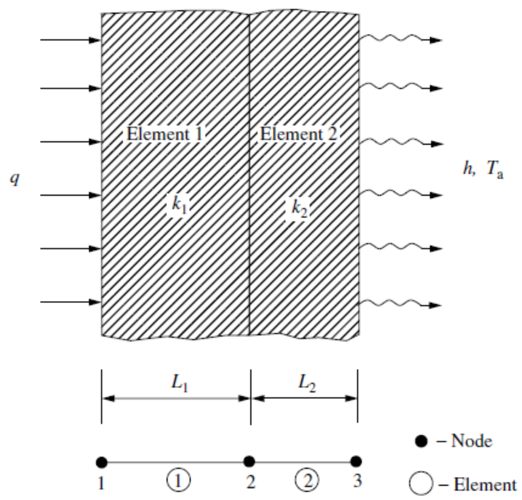


Fig. 3

Q. 5A Illustrate with three examples the use of Euler- Lagrange Equation for deducing GDE's of physical systems. (03)

Q.5B Derive the transient one dimensional heat conduction with usual notation, given by the following thermal equilibrium finite element equation, (07)

$$\frac{\rho c_p l A}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{Bmatrix}\frac{\partial T_i}{\partial t}\\\frac{\partial T_j}{\partial t}\end{Bmatrix} + \left(\frac{Ak_x}{l}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + \frac{hPl}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\right)\begin{Bmatrix}T_i\\T_j\end{Bmatrix}$$

$$= \frac{GAl}{2}\begin{Bmatrix}1\\1\end{Bmatrix} - \frac{qPl}{2}\begin{Bmatrix}1\\1\end{Bmatrix} + \frac{hT_a Pl}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$$

Q.6A For the steady state radial heat conduction in a hollow cylinder, show with usual notations that the thermal conductance matrix and thermal load vectors are given respectively by, (07)

$$[K] = \frac{2\pi k}{l}\left[\frac{(r_i + r_j)}{2}\right]\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + 2\pi r_0 h\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

$$\{f\} = hT_a 2\pi r_0\begin{Bmatrix}0\\1\end{Bmatrix}$$

Q.6B Derive the shape functions for a heat conducting Isoparametric 4 noded linear Quadrilateral element using the Serendipity approach (03)