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MANIPAL INSTITUTE OF TECHNOLOGY
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I SEM M. Tech. (CAMDA) DEGREE END SEMESTER EXAMINATIONS
NOVEMBER/DECEMBER 2015

SUBJECT: SOLID MECHANICS (MME 505)
REVISED CREDIT SYSTEM
(05/12/2015)

Time: 3 Hours. Max. Marks: 50

Instructions to Candidates:
- Answer **ANY FIVE FULL** questions.
- Missing data, if any, may be assumed appropriately.

1. a) Derive the differential equations of equilibrium in cylindrical coordinates, giving the conditions to be satisfied by the stress components when they vary from point to point. **(05)**

   b) For the state of stress at a point in a solid characterized by the components, **(05)**

   σ_x = 12.31 MPa; σ_y = 8.96 MPa; σ_z = 4.34 MPa

   τ_{xy} = 4.20MPa; τ_{yz} = 5.27MPa; τ_{zx} = 0.84MPa

   Find the values of principal stresses and their directions.

2. a) Obtain the stress-strain relations for linear isotropic materials obeying Hooke's law in the following form: **(06)**

$$\varepsilon_{xx} = \frac{1}{E}[\sigma_x - \vartheta(\sigma_y + \sigma_z)]$$

$$\varepsilon_{yy} = \frac{1}{E}[\sigma_y - \vartheta(\sigma_x + \sigma_z)]$$

$$\varepsilon_{zz} = \frac{1}{E}[\sigma_z - \vartheta(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}; \quad \gamma_{yz} = \frac{\tau_{yz}}{G} \quad and\ \gamma_{xz} = \frac{\tau_{xz}}{G}$$

   Where ε_{xx}, ε_{yy} and ε_{zz} are linear strains in ***x, y*** and ***z*** directions respectively γ_{xy}, γ_{yz} and γ_{xz} are shear strains in ***xy, yz*** and ***xz*** planes respectively**.**

σ_x, σ_y and σ_z are normal stresses on ***x, y*** and ***z*** planes respectively

τ_{xy}, τ_{yz} and τ_{xz} are shear stresses in ***xy, yz*** and ***xz*** planes respectively.

E and ϑ are modulus of elasticity and Poisson's ratio respectively.

b) A cylindrical rod is subjected to a torque ***T***. At any point ***P*** of the cross section, the following stresses occur **(04)**

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0;\ \tau_{yz} = G\theta x;\ \tau_{zx} = -G\theta y$$

Check whether these satisfy the equations of equilibrium. Also show that the lateral surface is free of load, i.e show that $T_x = T_y = T_z = 0$.

3. a) Derive the cubic equation which gives the state of principal strain at a point in a body in the form, **(05)**

$$\epsilon^3 - J_1\,\epsilon^2 + J_2\,\varepsilon - J_3 = 0$$

Where, J_1, J_2 and J_3 are the strain invariants..

b) Let $\sigma_x = -5c, \sigma_y = c, \sigma_z = c, \tau_{xy} = -c, \tau_{yz} = \tau_{zx} = 0$ where $c = 1000$ kPa. Determine the following: **(05)**

i) Maximum shear stress

ii) Octahedral stresses.

4. a) From the fundamental principles, obtain the Strain Energy function for a solid obeying Hooke's law and prove the statement "the partial derivative of the strain energy function with respect to force F_i, gives the displacement corresponding to F_i". **(06)**

b) Verify whether the following strain field satisfies the equations of compatibility, if p is a constant. **(04)**

$\varepsilon_{xx} = py$, $\varepsilon_{yy} = px$, $\varepsilon_{zz} = 2p(x + y)$

$\gamma_{xy} = p(x + y)$, $\gamma_{yz} = 2pz$ and $\gamma_{xz} = 2pz$

5. a) The displacement field for a body is given by

$$u = [\,(x^2 + y^2 + 2)i + (3x + 4y^2)j + (2x^3 + 4z)k\,] \times 10^{-4}$$

i) What is strain at P(1,2,3) in the direction of ***PQ*** having direction cosines $n_x = 0.6$, $n_y = 0$ and $n_z = 0.8$?

ii) What is the orientation of $P'Q'$ after deformation? **(06)**

b) Determine the diameter of a cold rolled steel shaft, 6 m long, used to transmit 35kW at 600 rpm. The shaft is simply supported at its ends in bearings. The shaft experiences bending due to its own weight also. Use a factor of safety 2. The tensile yield limit is 280 x 10^6 kPa and the shear yield limit is 140 x 10^6 kPa. Use the maximum shear stress theory. **(04)**

6. a) For steel following data are applicable: **(04)**

E = 207 x 10^6 kPa, G = 80 x 10^6 kPa and ***ϑ = 0.3***. For the given strain state at a point, determine the stress state and also evaluate Lame's coefficients.

$$[\epsilon_{ij}] = \begin{bmatrix} 32 & 0 & 160 \\ 0 & 864 & 24 \\ 160 & 24 & 240 \end{bmatrix} 10^{-3}$$

b) Discuss the following with respect to elastic strain energy stored in a body due to work done: **(06)**

i) Maximum Elastic Energy theory of failure

ii) Coressponding forces and displacements