Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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SEVENTH SEMESTER B.TECH (E & C) DEGREE END SEMESTER EXAMINATION NOV/DEC 2015

SUBJECT: INFORMATION THEORY AND CODING [ECE-407]

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates

- Answer **ANY FIVE** full questions.
- Missing data may be suitably assumed.

1A. Shortly before a horse-race, a book-maker believes that several horses entered in the race have the following probability of winning:

Horse	A	B	C	D	E
P(winning)	0.04	0.42	0.31	0.12	0.11

He, then receives a message that owing to a minor injury, one of the horses is not participating in the race. Explain how you would assess from an information theory point of view, the information value of this message.

a) If the horse in question is known

If it is not known

1B. A pair of dice are tossed and the outcome is recorded as (x_1, x_2) where x_1 is the outcome of the first dice and x_2 is that of the second dice.

a) Find the sample space

b) Find A and B, if A and B are two events defined as below.

$A = \{ (x_1, x_2) \text{ such that } x_1 + x_2 = 10 \}$

$B = \{ (x_1, x_2) \text{ such that } x_1 > x_2 \}$ then,

Find P(AB), P(B/A), P(A/B)

1C. In a facsimile transmission of picture, there are about 2.25×10^6 pixels/frame. For a good reproduction 12 brightness levels are necessary. Assume that all these levels are equally likely to occur. Find the rate of information if one picture is to be transmitted every 3 minutes. What is the source efficiency of this facsimile transmitter?

(5+3+2)

2A. Consider a first order markov source which has three states A, B and C. Let P(A/A)=P(B/B)=P(C/C)= 0.7. Let the probability of transition from a given state to any other state be equal to 0.15. Draw its state diagram. Find its entropy. Find the Adjoint of this source and its entropy, $H(\bar{S})$.

2B. Let S_0 be the third extension of a zero memory binary source with the probability of a 0 equal to p. Another source observes the output of S_0 and emits either a 0, 1, 2 or 3 according to whether the output of S_0 had 0,1, 2, 3 zeros.

(a) Compute $H(S_0)$

(b) Compute $H(S)$.

2C. Which of the sets of word lengths shown below are acceptable for uniquely decodable code when the code alphabet is X={0,1,2}.

(i) Code A with 11 symbols of word lengths 1,2,2,2,2,3,3,3,3,3,3

(ii) Code B with 11 symbols of word lengths 1,1,2,2,3,3,4,4,5,5,5

(5+3+2)

3A. The source S has nine symbols; each occurs with the probability 1/9.
a) Find a compact code for S using the code alphabet X = {0, 1, 2}
b) Find a compact code for S using the code alphabet X = {0, 1, 2, 3}
Also find average length for each case.

3B. Consider a second order markov source having symbol set $S = \{0,1\}$ with transition probabilities as P(0/00) = P(1/11) = 0.6, P(1/00) = P(0/11) = 0.4 and P(0/01) = P(1/01) = P(0/10) = P(1/10) = 0.5. Write the state diagram and find its entropy.

3C. Determine whether the following codes are instantaneously decodable:
(a) {0, 01, 11,111}
(b) {0, 01, 110,111}
(c) {0, 10, 110,111}
(d) {1, 10, 110, 111}

(5+3+2)

4A. Encode "**google**", using adaptive Huffman coding algorithm for the source with 26 letter alphabet (a to z).

4B. Design a ternary instantaneous code using Shannon-Fano coding algorithm for a source with source probabilities $P = \{0.3, 0.3, 0.12, 0.12, 0.12, 0.06, 0.06, 0.04\}$. Compute entropy and redundancy of the code.

4C. Prove that the average length L of an r-ary compact code for a zero memory source with q symbols, can never be less than its entropy.

(5+3+2)

5A. Consider a Binary symmetric communication channel with the probability of error 0.1, whose input source is the alphabet A = {0,1} with probabilities {0.6, 0.4} whose output alphabet are B = {0,1} and C={0,1}. Compute I(A;B,C).

5B. Compute capacity of of a binary information channel, $\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$.

5C. For the channel with the channel matrix given below and input symbols $a_1, a_2,$ and a_3 with probabilities 0.4, 0.3 and 0.3 respectively. Identify the maximum likelihood decision rule and compute probability of error for the same.

(5+3+2)

6A. Consider a systematic (n,k) code whose parity check equations are $v_0 = u_0 + u_1 + u_2$,

$v_1 = u_1 + u_2 + u_3$, $v_2 = u_0 + u_2 + u_3$, $v_3 = u_0 + u_1 + u_3$, where u_0, u_1, u_2 and u_3 are message digits (u_3 is the least significant digit) and v_0, v_1, v_2 and v_3 are parity check digits. Develop the generator and the parity check matrices for this code. List all code words.

6B. The (7, 4) linear block code with the parity check matrix given by

$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$. Develop a decoding circuit for this code.

6C. Find the capacity in bits, of the channel whose channel matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \overline{p} & p \\ 0 & p & \overline{p} \end{bmatrix}$$

(5+3+2)