



Manipal Institute of Technology, Manipal

(A Constituent Institute of Manipal University)



(08)

## SEVENTH SEMESTER B.TECH (INSTRUMENTATION & CONTROL ENGINEERING) END SEMESTER EXAMINATIONS, NOV/DEC 2015

SUBJECT: ROBUST CONTROL [ICE 439]

Time: 3 Hours

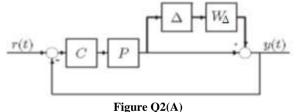
MAX. MARKS: 50

## Instructions to Candidates:

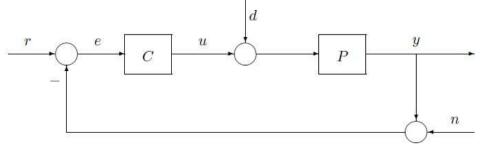
- \* Answer **ANY FIVE FULL** questions.
- ✤ Missing data may be suitably assumed.
- **1A.** Determine  $\| \|_{\infty}$  and  $\| \|_{2}$  norms of the system

$$G(s) = \begin{bmatrix} \frac{2}{s+10} \\ \frac{20}{s+1} \end{bmatrix}$$

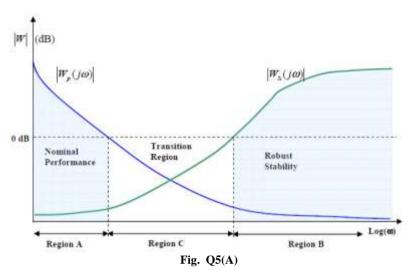
- **1B.** Consider nominal plant  $P(s) = \frac{1}{s+a_0}$  and actual plant model  $\tilde{P}(s) = \frac{1}{s+a}$  with a = (02)  $a_0 + \delta \delta_m$ ,  $|\delta| \le 1$ . Assume suitable weighting function W and rewrite uncertain system in the form  $\tilde{P} = P(1 + \Delta W)^{-1}$ , where  $\Delta$  is stable transfer function given by  $\Delta = \delta$  with  $|\delta| \le 1$ .
- 2A. Derive robust stability condition for the unity feedback control system as shown in (05) Fig. Q2(A)



**2B.** Assume that unity feedback system as shown in Fig. Q (2B) is internally stable (03) and n = d = 0. Show that if input r(t) is the ramp input then  $e(t) \rightarrow 0$  as  $t \rightarrow \infty$  if and only if sensitivity transfer function S(s) has at least two zero at the origin.



- **2C.** Illustrate sensitivity transfer function for the unity feedback control system and show (02) that  $||W_P S||_{\infty} < 1 \Leftrightarrow |W_P(j\omega)| < |1 + L(j\omega)|, \forall \omega$ .
- **3A.** Compute an internally stabilizing controller *C* for plant model  $P = \frac{1}{(s-1)(s-2)}$ . (06)
- **3B.** Show that set of all controllers for which the feedback system shown in Fig. Q(2B) is (04) internally stable is given by  $C = \left\{ \frac{X + MQ}{Y NQ} : Q \in R\mathcal{H}_{\infty} \right\}$
- **4.** State and prove small gain theorem with neat diagram
- **5A.** For the given weighting functions  $W_{\Delta}$  and  $W_{P}$ , derive the conditions on loop (07) function *L* for the regions *A*, *B* and *C* as shown in the Fig. Q5 (A) to satisfy robust performance inequality  $|||W_{p}S| + |W_{\Delta}T|||_{\infty} < 1$ .



**5B.** Show that  $|||W_PS| + |W_\Delta T|||_{\infty} < 1 \Rightarrow \max_{\omega} \overline{\sigma} \begin{bmatrix} W_PS \\ W_\Delta T \end{bmatrix} < \frac{1}{2}$  (03)

6. Consider the standard unity feedback loop as shown in Fig. Q2 (B) where r is the (10) reference input, d is the disturbance, n is the noise signal, P is the nominal plant and C is the controller which has to be designed.

If  $P(s) = \frac{1}{10s+1}$ , C(s) = K. Find the least positive gain K so that the followings are all hold:

- (i) The feedback system is internally stable
- (ii)  $|e(\infty)| \leq 0.1$  when r(t) is the unit step and n = d = 0,
- (iii)  $\|y\|_{\infty} \leq 0.1$  for all d(t) such that  $\|d\|_2 \leq 1$  when r = n = 0.

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