Reg. No.					



# Manipal Institute of Technology, Manipal



(A Constituent Institute of Manipal University)

# VII SEMESTER B.TECH (MECHANICAL ENGINEERING) END SEMESTER EXAMINATIONS, NOV/DEC 2015

## SUBJECT: MATHEMATICAL MODELLING ON MECHANICAL SYSTEMS [MME - 445]

### **REVISED CREDIT SYSTEM**

Time: 3 Hours

MAX. MARKS: 50

#### **Instructions to Candidates:**

- Answer ANY FIVE FULL the questions.
- Missing data may be suitable assumed.
- 1A Develop the Mathematical model for population growth. The annual birth and death rates in a country are 15.7% and 12.78% respectively, while the annual immigration and migration rates are 22.23%, 25.6%. Assuming the rates to be constant over a period of five years, use difference equation to formulate a model for population change and predict the populations of the next five years, if the current population is 685686.
- 1B With suitable assumptions develop SIS model for epidemic and obtain the function for S(t)
- 1C A bacterial culture contains two strains A and B, of bacteria, with respect to population of 10 million and 16 million initially. Each strain secretes a chemical that is toxic to the other, so that in an hour, each 3 bacteria of strain A kill one bacterium of strain B and each 6 bacteria of strain B kill one bacterium of strain A. Formulate a mathematical model using difference equations. Which strain will servile and how long will it take for the other to get wiped out?

(3+3+4)

A sky driver equipped with parachute and other essential equipment falls from rest toward the earth. The total weight of the man plus the equipment is 160 lb. Before the parachute opens, the air resistance is numerically equal to  $\frac{1}{2}v$ , where 'v' be the velocity. The parachute opens 5 sec after the fall begins, after it opens, the air resistance is numerically equal to  $\frac{5}{8}v^2$  Find the velocity i) before the parachute opens ii) after the parachute opens.

- 2B Define transient state of Markov chain. A salesman's territory consists of 3 cities A, B and C. He never sells in the same city of two consecutive days. If he sells in a city A, then the next day he sells in city B. However if he sells in either B or C, then the next day he is twice as likely to sell in city A as in the other city. In the long run how often does he sell in each of the cities?
- 2C For the two armies X and Y, the X army is about to attack Y army which has only 5000 troops while the X army has 10000 troops. The Y army however superior military equipment's which make each Y soldier 1.5 times as effective as X soldier. Develop a mathematical model by using differential equation and explain for i) which army will win ii) To estimate how many troops of the winning army will be left at the end.

$$(3+3+4)$$

- 3A With suitable assumptions explain the Harrod model in economics and finance.
- 3B The production planner at a factory manufacturing lawn cutters has advise the management on how many of each model to produce per week in order to maximize the profit. The factory produces two types of lawn cutters Quadrant and Pentagon. Each must go through two processes, body work and engine work and for more than 480 hours on body work. One quadrant requires 30 minutes of work and 30 minutes of body work. One pentagon requires 20 minutes of engine work and 12 minutes of body work. The ratio of Quadrants to Pentagons produce per week must be at least 2:3. A minimum of 200 Quadrants must be produced per week. Each Quadrant gives a profit of Rs 1200 and each Pentagon Rs 400. Formulate as linear programming model and sole graphically.
- 3C Use simplex method to minimize P = x 3y + 2z, subject to the constraints  $3x y + 2z \le 7$ ,  $-2x + 4y \le 12$ ,  $-4x + 3y + 8z \le 10$ ,  $x \ge 0$ ,  $y \ge 0$ ,  $z \ge 0$

(2+3+5)

- 4A Develop the mathematical model for diffusion of blood glucose in the blood stream
- 4B A police report provide the following facts: Police arrived at the scene of murder at 8 am. They immediately took and recorded the temperature of the corpse, which was 33° C and thoroughly inspected the area. By the time they finished inspection, it was 10 am. They again took the temperature of the corps, which has dropped to 29° C, and had the corpse sent to the morgue. The temperature at the crime scene had remained steady at 23° C. What is the time of death by assuming a normal body temperature of 37° C at this time? Model the situation using differential equations
- 4C A Lake contains 20000 fish at present. If there was no fishing the population of fish would increases by 20% every year. It is proposed to allow fishing at rate of 5000 fish per year. Suppose the lake contains 10000 m<sup>3</sup> of water with 4% pollution by volume. Every day 1000 m<sup>3</sup> of clean water flows into the lake and 1000 m<sup>3</sup> of polluted water flow out. How long it will take for the pollution in the lake to drop to a safe level of 2% and how long does it take the pollution of fish decreases to ze

(3+3+4)

5A Solve the following pay off matrix by using dominance rule. Determine the optimal strategies and value of the game.

$$PLB \\ PLA \begin{pmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{pmatrix}$$

- 5B Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number 'x' of infected students but also to the number of students not infected, determine the number of infected students after 6 days it is further observed that after 4 days x(4) = 50
- 5C Explain prey predator model and discuss the equilibrium values for this model.

(3 + 3 + 4)

6A Solve the Game by using linear programming. The payoff matrix for player A is given by *PLB* 

 $PLA \begin{pmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{pmatrix}$ 

6B A large vat holds 100 gallons of water that is to be mixed with sugar and then used to make soft drinks. Sugar-water containing 5 tablespoons of sugar per gallon enters the vat through a pipe at a rate of 2 gallon per minute. Another pipe pumps sugar-water with 10 tablespoons of sugar per gallon into the vat at rate of 1 gallon per minute. The vat is kept well mixed, so that the concentration of sugar in the vat is uniform. Sugar –water is drained out of the vat at rate of 3 gallon per minute. Find the amount of sugar in the vat at time t if the vat initially has 900 tablespoons init.

\*\*\*\*\*\*

(5 + 5)