Reg. No.					



# Manipal Institute of Technology, Manipal



(A Constituent Institute of Manipal University)

# VII SEMESTER B.TECH (MECHANICAL ENGINEERING) END SEMESTER MAKEUP EXAMINATIONS, DEC 2015/JAN 2016

## SUBJECT: MATHEMATICAL MODELLING ON MECHANICAL SYSTEMS [MME 445]

### **REVISED CREDIT SYSTEM**

Time: 3 Hours

MAX. MARKS: 50

#### Instructions to Candidates:

- Answer **ANY FIVE FULL** the questions.
- Missing data may be suitable assumed.
- 1A A system consists of n components, each of volume V stacked one on top of another. A solution with constant concentration C flows continuously into the topmost compartment at uniform rate R litres per minute, and the contents of each component flow into the immediately lower compartment at the same uniform rate. Formulate a mathematical model for determining the respective concentrations  $C_1, C_2...C_n$  of the solutions in the n components at any time.
- 1B Explain the logistic law of population growth.
- 1C A farmer has a 100 acre farm. He can sell all the tomatoes, potatoes and beans he can raise the price to obtain Rs 1 per kg for tomatoes, Rs 0.75 per kg of potatoes and Rs 2 per kg for beans. The average yield per acre is 2000 kg of tomatoes, 3000 kg of potatoes and 1000 kg of beans. Fertilizers are available at Rs 0.50 per kg and amount required per acre is 100 kg to each of tomatoes and potatoes and about 50 kg for beans. Labour required for cultivating and harvesting per acre is 5 man day for tomatoes and beans also 6 man days for potatoes.. A total of 400 manday of labour are available at Rs 20 per man-day. Formulate this problem as Linear programming model and maximize the former total profit by using graphical method.

#### (3+3+4)

- 2A The rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. Half of the original number of radioactive nuclei has undergone disintegration in a period of 1500 years.
  - a) What percentage of the original radioactive nuclei will remain after 4500 years
  - b) In how many years will only one tenth of the original number remain?

- 2B Define the equilibrium value of the dynamical system  $a_{n+1} = f(a_n)$ . Find the solution of the difference equation  $y_{n+2} 2\cos \alpha y_{n+1} + y_n = 1000$
- 2C A bacterial culture contains two strains A and B, of bacteria, with respect to population of 10 million and 16 million initially. Each strain secretes a chemical that is toxic to the other, so that in an hour, each 3 bacteria of strain A kill one bacterium of strain B and each 6 bacteria of strain B kill one bacterium of strain A. Formulate a mathematical model using differential equations. Which strain will servile and how long will it take for the other to get wiped out?

$$(3 + 3 + 4)$$

- 3A Explain twelve point procedures for solving problems through Mathematical Modelling.
- 3B Obtain the solution of Gambler's Ruin problems in probability theory.
- 3C A certain chemical is converted into another chemical by a chemical reaction. The rate at which the first chemical is converted is proportional to the amount of this chemical present at any instant. Ten percent of the original amount of the first chemical has been converted in 5 min. What percent of the first chemical will have been converted in 20 min? In how minutes will 60% of the first chemical have been converted?

(3 + 3 + 4)

- 4A The annual birth and death rates in a country are 11.7% and 10.78% respectively, while the annual immigration and emigration rates are 20.23% and 14.95%. Assuming the rates to be constant over a period of five years, use difference equation to formulate a model for population change and predict the populations of the next five years, if the current population is 368250
- 4B Thirty years ago your parents purchased a home by financing Rs100000 mortgage the charges 0.5% interest in each month. Formulate the Model in terms of monthly payment p that allows the mortgage to paid off after 360 payments
- 4C A police report provide the following facts: Police arrived at the scene of murder at 8 am. They immediately took and recorded the temperature of the corpse, which was 33° C and thoroughly inspected the area. By the time they finished inspection, it was 10 am. They again took the temperature of the corps, which has dropped to 29° C, and had the corpse sent to the morgue . The temperature at the crime scene had remained steady at 23° C. What is the time of death by assuming a normal body temperature of 37° C at this time? Model the situation using differential equations.

(3 + 3 + 4)

5A Solve the following pay off matrix by using dominance rule. Determine the optimal strategies and value of the game.

$$PLB$$

$$PLA\begin{bmatrix} 5 & -10 & 9 & 0 \\ 6 & 7 & 8 & 1 \\ 8 & 7 & 15 & 1 \\ 8 & 4 & -1 & 4 \end{bmatrix}$$

- 5B Two boys and two girls are throwing ball from one to other. Each boy throws the ball to the other boy with probability 1/2and each girl with probability 1/4. On the other hand each girl throws the ball to each boy with probability 1/2 and never the other girl. In the long run how often does each receive the ball?
- 5C Develop a Mathematical model for epidemic in which there are no removal and 'n' be the initial number of susceptible in which ten person are infected. Interpret the result for I(t) and S(t) as t tends to infinity.

(3 + 3 + 4)

- 6A Use Simplex method to minimize P = x 3y + 2z, subjected to the constraints:  $3x - y + 2z \le 7$ ,  $-2x + 4y \le 12$ ,  $-4x + 3y + 8z \le 10$ ,  $x \ge 0$ ,  $y \ge 0$ ,  $z \ge 0$
- 6B A lake contains 20000 fish at present. If there was no fishing the population of fish would increases by 20% every year. It is proposed to allow fishing at rate of 5000 fish per year. Suppose the lake contains 10000 m<sup>3</sup> of water with 4% pollution by volume. Every day 1000 m<sup>3</sup> of clean water flows into the lake and 1000 m<sup>3</sup> of polluted water flow out. How long it will take for the pollution in the lake to drop to a safe level of 2% and how long does it take the pollution of fish decreases to zero. (6 + 4)

\*\*\*\*\*\*