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MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER M.Sc. (APPLIED MATHEMATICS AND COMPUTING) DEGREE END SEMESTER MAKE-UP EXAMINATION- DEC 2016

SUBJECT: ALGEBRA (MAT 603)

Time: 3 Hrs. 29 December 2016 **Maximum Marks: 50**

Note: a) Answer any FIVE full questions b) All questions carry equal marks

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1A. Prove that a subgroup of a cyclic group is cyclic. Find all subgroups of Z_{18}.

1B. Let H be a subgroup of a finite group G. Then prove that the order of H is a divisor of the order of G.

1C. State and prove division algorithm for polynomial over a field.

(3 + 3+ 4)

2A. Define integral domain. Give an example. Verify whether $Z \times Z$ an integral domain? Justify?

2B. If R is a ring with unity, and I is an ideal of R containing unit, then prove that I = R.

2C. If G is a finite group and p is prime number, then prove that any two p-Sylow subgroups of G are conjugate.

(3 + 3+ 4)

3A. Define primitive polynomial. Prove that the product of two primitive polynomials is also a primitive polynomial.

3B. Prove that the ring Z[i] of Gaussian integers is a Euclidean domain.

3C. Apply Eisenstein criteria for irreducibility to test whether the polynomials: $f(x) = 8x^3 + 6x^2 - 9x + 24$, $g(x) = 4x^{10} + 9x^3 + 24x - 18$ are irreducible over Q or not.

(3 + 3+ 4)

4A. If p is a prime number and p divides o(G) then prove that G has an element of order p.

4B. Prove that any group of order of less than or equal to 5 is abelian.

4C. Let X be a G-set. Prove that for each $g \in G$, the function $\sigma_g: X \rightarrow X$ defined by $\sigma_g(x) = gx$ for $x \in X$ is a permutation of X. Also, prove that the map $\phi: G \rightarrow S_x$ defined by $\phi(g) = \sigma_g$ is a homomorphism with the property that $\phi(g)(x) = gx$.

(3 + 3+ 4)

5A. Let X be a G-set. Then prove that G_x is a subgroup of G for each $x \in X$.

5B. Find order of the given element in the direct product:

(i) (1, 5) in $Z_3 \times Z_6$

(ii) (3, 6) in $Z_4 \times Z_{12}$

(iii) (1, 2, 8, 16) in $Z_4 \times Z_{12} \times Z_{20} \times Z_{24}$

5C. Classify the group $(Z_2 \times Z_4)/\langle(0, 1)\rangle$ according to the fundamental theorem of finitely generated abelian groups.

(3 + 3 + 4)

6A. Suppose M and N are submodules of a module P over R. Then prove that $M \cap N = (0)$ if and only if every element $z \in M + N$ can be uniquely written as $z = x + y$ with $x \in M$ and $y \in N$.

6B. Define submodule. Prove that unitary modules over Z are simply abelian groups.

6C. Prove that a field F contains a subfield isomorphic to either Z_p where p is a certain prime integer, or the field of rational number.

(3 + 3+ 4)