R.No.										

MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER M.Sc. (APPLIED MATHEMATICS AND COMPUTING) DEGREE
END SEMESTER EXAMINATION- NOV/DEC 2016
SUBJECT: ALGEBRA (MAT 603)

Time: 3 Hrs. 25 November 2016 **Maximum Marks: 50**

Note: a) Answer any FIVE full questions b) All questions carry equal marks

1A. Define cyclic group. Prove that if $a \in G$, the set $H = \{a^n \mid a \in Z\}$ is a subgroup of G and is the smallest subgroup of G contains a.

1B. Prove that every group is isomorphic to a subgroup of A(S) for some appropriate set S.

1C. Define Euclidean domain and give an example of Euclidean domain. State and prove unique factorization theorem.

(3 + 3+ 4)

2A. Prove that a finite integral domain is a field. Verify whether $(Z_{12}, +, \cdot)$ an integral domain.

2B. If R is a commutative ring with unit element and P is an ideal of R, then P is prime if and only if R/P is an integral domain. What are the prime ideals in the ring of integers?

2C. State and prove third part of Sylow's theorem.

(3 + 3+ 4)

3A. State and prove Gauss lemma for polynomial rings over rational field.

3B. Let f be a homomorphism of group G onto group G^1 with kernel K. Prove that kernal is a normal subgroup and show that $G/K \cong G^1$.

3C. State the Eisenstein criteria for irreducibility. Verify the irreducibility of the polynomial $25x^5 - 9x^4 + 3x^2 - 12$ over rational filed. Show that $x^2 + 1$ is irreducible in $Z_3[x]$.

(3 + 3+ 4)

4A. If G is a finite group, then prove that $C_a = \frac{O(G)}{O(N(a))} = i(N(a))$.

4B. Let G be a group of order pq where p and q are primes and $p > q$. If $a \in G$ is of order p and A is the subgroup of G generated by a, then prove that A is a normal subgroup of G.

4C. Let G be a group. Define $a * x = axa^{-1}$ for all $a, x \in G$. Verify whether G is a G-set? Also, prove that G_x with usual notation, is a subgroup of G for each $x \in X$, where X is a G-set.

(3 + 3+ 4)

5A. Define group action on a set. Let G be a finite group and X a finite G-set. If r is the number of orbits in X under G, then prove that $r \cdot |G| = \sum_{g\in G}|X_g.|$

5B. Find order of the given element in the direct product:

(i) (2, 4) in $Z_4 \times Z_{12}$

(ii) (3, 6) in $Z_6 \times Z_{15}$

(iii) (2, 6, 12, 16) in $Z_4 \times Z_{12} \times Z_{20} \times Z_{24}$

5C. Find the order of $(2, 1) + \langle(1, 1)\rangle$ in $(Z_3 \times Z_6)/\langle(1, 1)\rangle$. Let G be a finitely generated abelian group with generating $\{a_1, a_2, \ldots, a_n\}$. Show that there is a homomorphism of $Z \times Z \times \ldots \times Z$ onto G.

(3 + 3 + 4)

6A. Prove that any two p-Sylow subgroups of G are conjugate.

6B. Prove that M is a left R- module if and only if there exists a homomorphism of rings R and $End_Z(M)$.

6C. Let R be an integral domain and M be an R module. Prove that the subset $T \subseteq M$ of all torsion elements is a submodule of M such that M/T is torsion free.

(3 + 3+ 4)