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# MANIPAL UNIVERSITY, MANIPAL

**FIRST SEMESTER M.SC (APPLIED MATHEMATICS & COMPUTING) END SEMESTER EXAMINATION NOV /DEC 2016**

**SUB: DIFFERENTIAL EQUATIONS (MAT - 601)**
**(REVISED CREDIT SYSTEM)**

**Time: 3 Hrs.** **23/11/2016** **Max.Marks: 50**

**Note: a) Answer any FIVE full questions and they carry equal marks (4 +3+3)**

1A. Let $\phi_1, \phi_2, \dots, \phi_n$ be solutions of $L(y)=y^n + a_1 y^{n-1} + \dots + a_n y=0$ on an interval I and $\boldsymbol{x_0} \in \boldsymbol{I}$ Then prove that $W(\phi_1, \phi_2, \dots, \phi_n)(x) = W(\phi_1, \phi_2, \dots, \phi_n)(x_0)\boldsymbol{e^{-a_1(x-x_0)}}$ where $\boldsymbol{a_1}$ is the coefficient of $y^{n-1}$.

1B. Prove that

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$
$$H_n'(x) = 2\,n\,H_{n-1}(x)$$

1C Prove that $\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2-1}$

2A. State and prove **uniqueness** theorem on n th order linear differential equation L(y) = 0 with constants coefficients and usual initial conditions.

2B Solve in series $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$.

2C. Prove that $\int_0^x [tJ_n^2(t)]dt = \frac{x^2}{2}[J_n^2(x) - J_{n-1}(x).J_{n+1}(x)]$

3A. Solve $y'' - 4y = 3e^{2x} + 4e^{-x}$ by annihilator method.

3B. Obtain the solution of Bessel's differential equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ when n is an integer.

3C State and prove **existence** theorem on n th order linear differential equation L(y) = 0 with variable coefficients and usual initial conditions.

4A. .(i) Show that the functions $\varphi_1$, $\varphi_2$, defined by $\varphi_1 = x^2$, $\varphi_2 = x\,|\,x\,|$, are linearly independent for $-\infty < x < \infty$.

(ii) Compute the Wronskian of these functions. Compare the two results and explain your answer.

4B Prove that $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = \begin{cases} 0 & , \quad \alpha \neq \beta \\ \frac{1}{2}\left[J_{n+1}(\alpha)\right]^2 & , \quad \alpha = \beta \end{cases}$ where $\alpha$, $\beta$ are the roots of $J_n(x) = 0$.

4C Prove that n solutions of $L(y) = 0$ on an interval I are linearly independent if ,and only if , Wronskian never equals to zero.

5A. Show that every solution of the constant coefficient equation $y'' + a_1 y' + a_2 y = 0$ tend to to zero as x tends to infinity if, and only if, the real parts of the roots of the characteristic polynomial are negative.

5B. Suppose $\phi$ is a function having continuous derivative on $0 \leq x < \infty$ such that $\phi'(x) + 2\phi(x) \leq 1$ for all x and $\phi(0) = 0$. Show that $\phi(x) \leq \frac{1}{2}$ for $x \geq 0$.

5C. Prove that $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}\left[\left(x^2-1\right)^n\right]$

6A. State and prove orthogonal property of Hermite functions.

6B. Solve by method of variation of parameters $y''' + y'' + y' + y = 1$

6C. (i) Prove that $L_n(x) = e^x \frac{d^n}{dx^n}\left(x^n e^{-x}\right)$.

(ii) Show that eigen functions belonging to two different eigen values are orthogonal with respect to r(x) in ( a,b)