Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

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A Constituent Institution of Manipal University

I M.Sc. (Applied Mathematics and Computing)

END SEMESTER MAKEUP EXAMINATIONS, NOV/ DEC 2016

SUBJECT: REAL ANALYSIS [MAT 605]

REVISED CREDIT SYSTEM

(30/12/2016)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **any FIVE full** questions.

1A.	Define a compact set. Show that compact subsets of metric spaces are closed.	**3**
1B.	Show that A set E is closed iff its complement is open	**3**
1C.	Show that the set of real numbers R has Archimedean property and hence show that there exists a rational number between any two real numbers.	**4**
2A.	Let $\{p_n\}$ be a sequence in a metric space X. If $p \in X$, $p' \in X$, and if $\{p_n\}$ converges to p and to p′, then show that p′ = p.	**3**
2B.	If p > 0, then show that $\lim_{n \to \infty} \sqrt[n]{p} = 1.$	**3**
2C.	If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of every finite sub collection of $\{K_\alpha\}$ is nonempty, then show that $\bigcap K_\alpha$ is non empty.	**4**
3A.	Obtain the radius of convergence and circle of convergence for (i) $\sum_{n=0}^{\infty} \left(\frac{2}{n}\right)^n z^n$ (ii) $\sum_{n=0}^{\infty} \left(1-\frac{3}{n}\right)^{n^2} z^n$	**3**
3B.	If f is a continuous mapping of a compact metric space X into a metric space Y, then show that f(X) is compact.	**3**
3C.	Let $e = \sum_{n=0}^{\infty} \frac{1}{n!}.$ Show that 2 < e < 3. Also show that e is irrational.	**4**

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4A.	If f is a continuous mapping of a compact metric space X into a metric space Y, then show that f is uniformly continuous.	**3**
4B.	Let [a, b] be a given interval. Suppose that f is a bounded function on [a, b] and α is a monotonically increasing function on [a, b]. Show that $\underline{\int_{a}^{b}} f d\alpha \le \overline{\int_{a}^{b}} f d\alpha.$	**3**
4C.	If a partition P^* is a refinement of a partition P of [a, b], f is a bounded function on [a, b] and α is a monotonically increasing function on [a, b], then show that $L(P, f, \alpha) \le L(P^*, f, \alpha)$	**4**
5A.	Suppose K is compact, and (i) $\{f_n\}$ is a sequence of continuous functions on K, (ii) $\{f_n\}$ converges point wise to a continuous function f on K, (iii) $f_n(x) \ge f_{n+1}(x)$ for all $x \in K$, n = 1, 2, 3, ….. then show that $f_n \to f$ uniformly on K.	**3**
5B.	Let $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and let $\lim_{t \to x} f_n(t) = A_n \quad (n = 1, 2, 3......)$ then show that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$	**3**
5C.	If X is a complete metric space, and if ϕ is a contraction of X into X, then show that there exists one and only one $x \in X$ such that $\phi(x) = x$.	**4**
6A	Show that the sequence of functions $\{f_n\}$ defined on E, converges uniformly on E if and only if for every $\varepsilon > 0$, there exists an integer N such that $m \ge N$, $n \ge N$, $x \in E$ implies $\lvert f_n(x) - f_m(x) \rvert < \varepsilon$	**4**
6B	Let $\{E_\alpha\}$ be a collection of sets E_α. Show that $\left(\bigcup_\alpha E_\alpha \right)^C = \bigcap_\alpha E_\alpha^C$	**3**
6C	If $f \in R(\alpha)$ and $g \in R(\alpha)$ on [a, b], then show that $fg \in R(\alpha)$.	**3**