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MANIPAL UNIVERSITY, MANIPAL

I M.Sc. (Applied Mathematics and Computing)

END SEMESTER EXAMINATIONS, NOV/DEC 2016

SUBJECT: REAL ANALYSIS [MAT 605]

REVISED CREDIT SYSTEM
(28/11/2016)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer any **FIVE** full questions.

1A.	Define a compact set. If F is a closed subset and K is compact subset of a metric space X, show that F$\cap$K is compact.	**3**
1B.	Show that every neighbourhood is an open set.	**3**
1C.	Show that the set of real numbers R has Archimedean property and hence show that there exists a rational number between any two real numbers.	**4**
2A.	Define a complete metric space. In any metric space X show that every convergent sequence is a Cauchy sequence.	**3**
2B.	If p > 0 and α is real, then show that $\lim\limits_{n\to\infty} \frac{n^{\alpha}}{(1+p)^n} = 0$.	**3**
2C.	Define a connected set. Show that a subset E of the real line R is connected if and only if it has the following property: If x $\in$E, y$\in$E, and x < z < y, then z $\in$E.	**4**
3A.	Obtain the radius of convergence and circle of convergence for (i) $\sum\limits_{n=0}^{\infty}\left(\frac{n+1}{n+2}\right)^n z^n$ (ii) $\sum\limits_{n=0}^{\infty}\frac{z^n}{n!}$	**3**
3B.	Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then show that f(X) is compact.	**3**

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3C.	Let $e=\sum_{n=0}^{\infty}\frac{1}{n!}$. Show that 2 < e < 3. Also show that e is irrational.	**4**
4A.	Show that a mapping of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.	**3**
4B.	Show that $f \in R(\alpha)$ on [a, b] if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.	**3**
4C.	If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on [a, b], then show that $f_1 + f_2 \in R(\alpha)$ and that $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.	**4**
5A.	State and prove Cauchy criterion for uniform convergence.	**3**
5B.	Suppose K is compact, and (i) $\{f_n\}$ is a sequence of continuous functions on K, (ii) $\{f_n\}$ converges point wise to a continuous function f on K, (iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, n = 1, 2, 3, ….. then show that $f_n \to f$ uniformly on K.	**3**
5C.	If X is a complete metric space, and if ϕ is a contraction of X into X, then show that there exists one and only one $x \in X$ such that $\phi(x) = x$.	**4**
6A.	Show that a set E is open iff its complement is closed.	**3**
6B.	Show that a countable union of countable sets is countable.	**3**
6C.	If a partition P^* is a refinement of a partition P of [a, b], f is a bounded function on [a, b] and α is a monotonically increasing function on [a, b], then show that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.	**4**