Reg.No.									

MANIPAL UNIVERSITY, MANIPAL - 576 104

THIRD SEMESTER M.Sc (APPLED MATHEMATICS & COMPUTING)

End Semester Makeup Examination Dec-2016

SUB: GRAPH THEORY (MAT 707)
(REVISED CREDIT SYSTEM)

Time: 3 Hrs. **Max. Marks:50**

Note: Answer any FIVE full questions.

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1. (a) Let G be a (p, q) graph all of whose vertices have degree k or $k + 1$. If G has $p_k > 0$ vertices of degree k and p_{k+1} vertices of degree $k + 1$, then show that $p_k = (k + 1)p - 2q$. (4)

   (b) Show that in any party with 6 people, either there are 3 people who mutually know each other or 3 people who mutually do not know each other. (3)

   (c) Show that every s.c. graphs has 4n or 4n+1 vertices for some positive integer n (3)

2. (a) With usual notation show that $k(G) \leq \lambda(G) \leq \delta(G)$. (4)

   (b) Show that every tree has a center consisting of either one vertex or two adjacent vertices. (3)

   (c) Show that a (p, q) graph is a tree if and only if it is acyclic and $p = q + 1$.(3)

3. (a) Let G be a (p, q)simple graph and let u and v be two non adjacent vertices of G such that $\deg(u) + \deg(v) \geq p$. Then show that G is Hamiltonian if and only if $G + (u, v)$ is Hamiltonian. (4)

   (b) Show that a graph is bipartite if and only if it has no odd cycle. (3)

   (c) Define line graph $L(G)$ and total graph $T(G)$ of a graph G. Find the number of edges in $L(G)$ and $T(G)$. (3)

4. (a) With usual notation show that $\alpha_0 + \beta_0 = \alpha_1 +_1 = p$ where p is the number of vertices in G. (4)

(b) Show that a matching in G is maximal if and only if G contains no augmenting path. (3)

(c) With usual notation show that for a planar (p,q) graph G, $p+q-r=2$ where r is number of regions. (3)

5. (a) Find the chromatic polynomial of the graphs C_4 and $K_{1,3}$. (4)

(b) Show that if G is a cycle on n vertices then the chromatic polynomial of G is given by $\pi_k(G) = (k-1)^n + (-1)^n(k-1)$ (3)

(c) Define Adjacency matrix $A(G)$ of a graph G. Show that $(i,j)^{th}$ entry of A^ngives the number of walks of length n between the vertices v_i and v_j in G. (3)

6. (a) Define incidence matrix of a graph G. Show that it is of rank $n-1$ when G has n vertices. (4)

(b) Let G be a graph with $0,1,-1$ incidence matrix $Q(G)$. Then show that $Q(G)$is totally unimodular. (3)

(c) Show that a graph G in which every vertex is of even degree cannot have a bridge. (3)

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