Reg.No.									

MANIPAL UNIVERSITY, MANIPAL - 576 104

THIRD SEMESTER M.Sc(APPLED MATHEMATICS & COMPUTING)

End Semester Examination Nov/Dec-2016

SUB: GRAPH THEORY (MAT 707)
(REVISED CREDIT SYSTEM)

Time: 3 Hrs. **Max. Marks:50**

Note: Answer any FIVE full questions.

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1. (a) With usual notation show that the Ramsey number $R(m,n)$ satisfies the relation $R(m,n) \leq R(m-1,n) + R(m,n-1)$. Hence show that $R(m,n) \leq {}^{m+n-2}C_{m-1}$. (4)

   (b) Let G_1be a (p_1, q_1) and G_2be a (p_2, q_2) graphs. Then find number of vertices and edges in (i) $G_1 \times G_2$ (ii) $G_1[G_2]$. (3)

   (c) With usual notation prove that $k(G) \leq \lambda(G) \leq \delta(G)$. (3)

2. (a) Let G be a (p,q) graph. Then show that the maximum value of when G does not contain any triangle is $\left[\frac{p^2}{4}\right]$. (4)

   (b) Show that a graph G with $p \geq 3$ vertices is 2-connected if and only if any two vertices of G are connected by atleast two internally disjoint paths. (3)

   (c) If G is a connected graph on $p(\geq 3)$ vertices with $\delta(G) \geq {}^{p}/_{2}$, then show that G is Hamiltonian. (3)

3. (a) Let W be the set of all vertices if degree $(p-1)$ in a graph G of order p, where p is even. Show that G has a perfect matching if the number of odd components of $(G-W)$ does not exceed $|W|$ and if every component of $(G-W)$ is complete. (4)

   (b) Let G be a bipartite graph with bipartition (X, Y). Then show that G contains a matching that saturates every vertex of X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. (3)

   (c) Show that the Ramsey number $r(k,k)$ satisfies the inequality

$r(k,k) \geq 2^{\frac{k}{2}}$. (3)

4. (a) Let G be a tree and $Q_n(G)$ be the reduced $0,1,-1$ incidence matrix obtained by deleting the last row of the $0,1,-1$ incidence matrix $Q(G)$. Let $P_n(G)$ be the path matrix where i^{th} column corresponds to the path vector of the path from the vertex i to the vertex n, where n is a fixed vertex corresponds to the last row of $Q(G)$. Then show that $P_n(G)$is inverse of $Q_n(G)$. (4)

(b) Show that the adjacency matrix of a bipartite graph is totally unimodular. (3)

(c) If G is simple graph then the chromatic polynomial $\pi_k(G)$ satisfies the relation $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$. And hence show that if G is a tree on n vertices, then $\pi_k(G) = k(k-1)^{n-1}$. (3)

5. (a) Let G be a graph and $S \subseteq V(G)$. Define complements of G. Show that every critical graph is a block. (4)

(b) Show that K_5 and $K_{3,3}$ are non planar. (3)

(c). Define the $0,1,-1$ incidence matrix $Q(G)$ of a graph G with an example. Show that, if G a (p,q) graph then rank of $Q(G)$ is $p-1$. (3)

6. (a) With usual notation show that

(i) $2\sqrt{p} \leq \chi + \bar{\chi} \leq p+1$

(ii) $p \leq \chi\bar{\chi} \leq \left(\frac{p+1}{2}\right)^2$. (4)

(b) show that a connected graph is isomorphic to its line graph if and only if it is a cycle. (3)

(c) For any (p,q) graph G with line graph $L(G)$, show that $A\big(L(G)\big) = B^T B - 2I_q$ where B is the $(0,1)$ incidence matrix of G and $A(L(G))$ is the adjacency matrix of $L(G)$. (3)

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