

Akhilash Ranjan

Time: 3 hours

Marks: 50

1. (i) Describe one natural phenomena which shows wave nature of particle. [5]

(ii) What is the difference between a wave equation and the Schrödinger equation? $[2\frac{1}{2}]$

(iii) What is the difference between orthogonality and orthonormality? $[2\frac{1}{2}]$

2. (i) A unitary transformation transforms one complete set of basis vectors into another. Prove that it also transforms the matrix representation of an operator with respect to one set into other. [5]

(ii) Prove that the fundamental commutation relation $[x, p_x] = i\hbar$ remains unchanged under unitary transformation. [5]

3. Show that for a finite deep square potential well only finite number of energy levels are possible. [10]

4. (i) A harmonic oscillator moves in a potential $V(x) = \frac{1}{2}kx^2 + cx$, where c is a constant. Find the energy eigenvalues. [5]

(ii) A positron and an electron form a shortlisted atom called positronium before the two annihilate to produce gamma rays. Calculate, in eV , the ground state energy positronium. [5]

5. (i) Formulate the Schroedinger equation for a free axis rigid

rotator. Write the expressions for energy eigenvalues and eigenfunctions. [5]

(ii) Find the number of energy levels in the range $E < \frac{15h^2}{8ma^2}$ of a cubical box of side a . [5]

6. (i) N noninteracting bosons are in an infinite deep potential well defined by $V(x) = \infty$ for $x < 0$ and for $x > a$. Find the ground state energy of the system. What would be the ground state energy if the particles are fermions? [5]

(ii) Show that if a wavefunction $\psi(1, 2, 3, \dots, n)$ is an energy eigenfunction of a symmetric Hamiltonian that corresponds to a nondegenerate eigenvalue, it is either symmetric or antisymmetric. [5]
