3A. Consider a mutual exclusion scenario where the concurrent processes share a resource (such as a file on a disk or a database entry), it is necessary to ensure that they do not have access to it at the same time. Define various properties for the proposed protocol, and write the LTL specifications for all the properties. If some of the properties can not be expressed in LTL, then give the rationale behind it. [5]

3B. Consider the Kripke model $\mathcal{M}$ of Figure Q.3B. Decide for each of the LTL formulae ϕ_i below, whether $\mathcal{M}, \models \phi_i$ holds. Justify your answer with suitable explanation.

i) $\phi_1 = FGc$

ii) $\phi_2 = GFc$

iii) $\phi_3 = X\neg c \rightarrow XXc$

iv) $\phi_4 = Ga$

v) $\phi_5 = aUG(b \vee c)$

vi) $\phi_6 = (XXb)U(b \vee c)$ [3]

3C. Run CTL labelling algorithm for the transition diagram given in Figure Q.3C to the formula $E[\neg c_1 \; U \; c_2]$. [2]

4A. Formalize the wise-men puzzle in the modal logic $KT45^n$. [5]

4B. Let $\mathcal{F} = (W, R)$ be a frame. The following statements are equivalent:

- R is transitive
- $\mathcal{F}$ satisfies $\Box\phi \rightarrow \Box\Box\phi$
- $\mathcal{F}$ satisfies $\Box p \rightarrow \Box\Box p$

Prove above statements using correspondence theory. [3]

4C. Find the natural deduction proofs for the following sequent over the basic modal logic K.

$$\vdash_K \Box(p \rightarrow q) \vdash \Box p \rightarrow \Box q$$

[2]

5A. With reference to NuSMV, explain the following

i) Inclusion operator

ii) Case expression

iii) If-Then-Else expression

iv) Basic next expression

v) Count operator [5]

5B. Describe different types of variable declaration in a finite state machine in the NuSMV language. [3]

5C. Write syntax of LTL formulas recognized by NuSMV. [2]

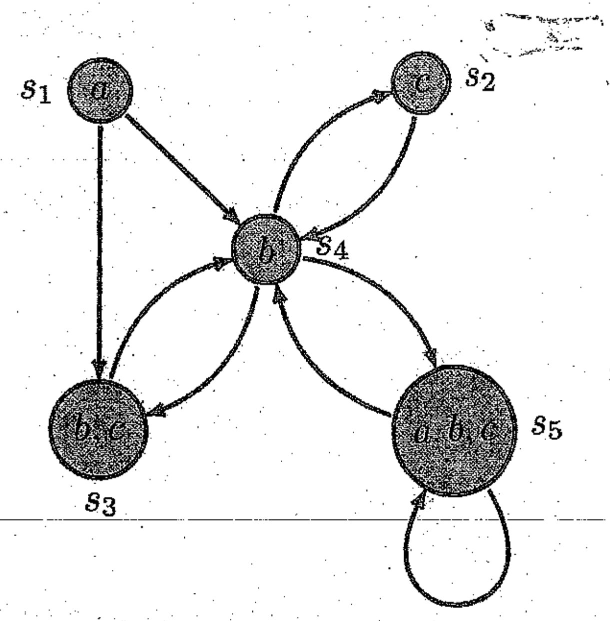

Figure: Q.3B

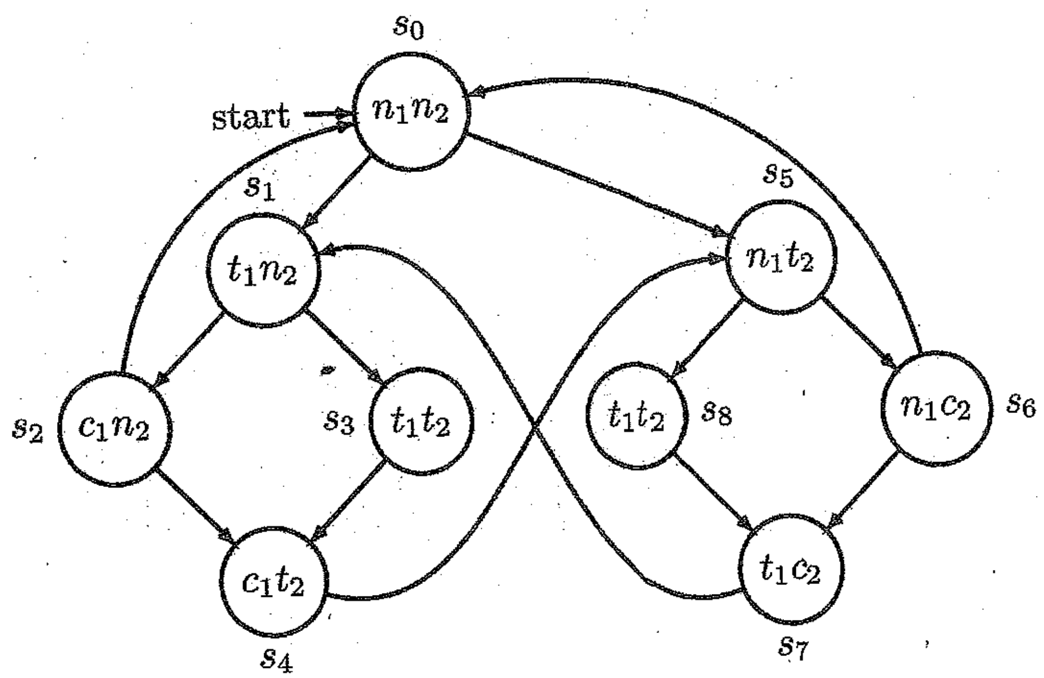

Figure: Q.3C

Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

A Constituent Institute of Manipal University, Manipal

FIRST SEMESTER M.TECH (Software Engineering) END SEMESTER DEGREE EXAMINATION-NOV/DEC 2016
SUBJECT: MATHEMATICAL LOGIC (ICT 5122)
(REVISED CREDIT SYSTEM)

TIME: 3 HOURS | **26/11/2016** | **MAX. MARKS: 50**

Instructions to candidates

- Answer **ALL** questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.

1A. Consider the argument "*If I were smart or good-looking, I would be happy and rich. But I am not rich. So it's true that either I'm not happy or I'm not rich. In other words, I am not both happy and rich. Therefore I am not smart or good-looking. In other words I am not smart and neither am I good-looking. In particular, I am not smart.*". Write a sequent in propositional logic capturing the logical structure of this argument, and prove the sequent using natural deduction rules. [5]

1B. Consider the sequent $p \vee q, p \to r \vdash r$. Determine a DAG which is not satisfiable iff this sequent is valid. Tag the DAG's root node with '1:T,' apply the forcing laws to it, and extract a witness to the DAG's satisfiability. Explain in what sense this witness serves as an explanation for the fact that $p \vee q, p \to r \vdash r$ is not valid. [3]

1C. Prove the validity of the sequent $(p \vee (q \to p)) \wedge q \vdash p$. [2]

2A. Consider the argument, "*Every student can swim. Everyone can either swim or surf. Betty can not swim. Therefore, not everyone who can surf is a student.*". Write a sequent in predicate logic for the given for the given sequent, and prove the sequent. [5]

2B. Prove the equivalence $(\exists x\phi) \vee (\exists x\psi) \dashv\vdash \exists x(\phi \vee \psi)$ in first-order logic. [3]

2C. Let $\mathcal{F} \stackrel{def}{=} \{e, \cdot\}$ and $\mathcal{P} \stackrel{def}{=} \{\leq\}$, where e is a constant, $\cdot$ is a function of two arguments and $\leq$ is a predicate in need of two arguments as well. Again, we write $\cdot$ and $\leq$ in infix notation as in $(t_1 . t_2) \leq (t.t)$. The model $\mathcal{M}$ has set A all binary strings, finite words over the alphabet $\{0, 1\}$ including the empty string ϵ. The interpretation $e^{\mathcal{M}}$ of e is just the empty word ϵ. The interpretation $\cdot^{\mathcal{M}}$ of $\cdot$ is the concatenation of words. Finally, we interpret $\leq$ as the prefix ordering of words. We say that s_1 is a prefix of s_2 if there is a binary word s_3 such that $s_1 \cdot^{\mathcal{M}} s_3$ equals s_2. Thus, $\leq^{\mathcal{M}}$ is the set $\{(s_1, s_2) | s_1 \text{ is a prefix of } s_2\}$. For the given model $\mathcal{M}$, check the satisfaction relation $\mathcal{M} \models \phi$, where ϕ :

i) $\forall x((x \leq x \cdot e) \wedge (x \cdot e \leq x))$

ii) $\neg\exists x \forall y((x \leq y) \to (y \leq x))$

[2]